CURRICULUM

Mathematics 436

Secondary School

Québec

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Secondary School

Direction de la formation générale des jeunes

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PAULINE MAROIS
Minister of Education

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Introduction

Mathematics 436 is a program designed for Secondary IV students who wish to study the sciences, commerce, business administration or a technical trade and who have demonstrated the ability to do so.

To prepare young Quebecers for the demanding world of the twenty-first century, schools must focus on the students' cognitive growth and the development of basic skills (i.e. communication and problem-solving skills as well as the ability to work with technology).

Because society is changing so rapidly, and owing to developments in the field of mathematics education, it is important to stress the interconnection of knowledge, skills and attitudes in the teaching of this program.

Mathematics 436 differs from *Mathematics 416* in two ways. First, it covers more material in greater detail and deals with more complex situations, problems and applications. Secondly, the students must use advanced terminology and formal notation, always be rigorous and precise, and justify every step in their solutions. In addition to preparing the students for science instruction, mathematics education should provide fertile ground for the development of skills that will be useful to them in the future: As Resnick and Klopfer have noted, "Graduates must not only be literate; they must also be competent thinkers."[1]

Three Major Guiding Principles

Current knowledge of the learning process and the focus of student learning have led to an emphasis on three principles intended to guide teachers in their work with students. These principles are as follows: to encourage the students to participate actively in the learning process, to encourage them to use a problem-solving approach at each stage of the learning process and to encourage them to use the appropriate technology for each task.

Encouraging Students to Take an Active Part in Their Own Learning Process

A great many research studies have shown that students should play a central role in their own learning process. In short, they should be ultimately responsible for their education:

1. L.B. Resnick and L.E. Klopfer, "Toward the Thinking Curriculum: An Overview," in *Toward the Thinking Curriculum: Current Cognitive Research, 1989 Yearbook of the Association for Supervision and Curriculum Development*, ed. Lauren B. Resnick and Leopold E. Klopfer (Alexandria, Va.: Association for Supervision and Curriculum Development, 1989), 1.

> The construction of a given concept is a complex process that depends first and foremost on the student. Concepts are not directly transmitted from a knowledgeable person to a student who supposedly knows nothing in a given field. Before they tackle new subject matter, students have already developed their own ideas, which are well organized, practical and sometimes fairly resistant to the changes targeted in a course of study.
>
> Thus, teaching involves creating situations in which students draw on their own knowledge. Teaching involves structuring the learning process around their strategies and thinking in order to try to get them to make progress in the construction of a given concept.[2]

To help students acquire the knowledge and skills targeted by this program, it is important to design learning situations that call upon their powers of observation and dexterity and that involve manipulations, exploration, construction and simulations. Through these activities, the students analyze hypotheses, actively look for solutions, discuss their approaches, analyze concepts or theories from their own point of view while taking into account other points of view, actively question the meaning and consequences of the procedures they use and relate the knowledge they have acquired to their own experience. These situations encourage the students to reflect, act, react and establish connections with what they have already learned.

Another way teachers can encourage students to participate in their learning process is by developing a suitable teaching approach. Teachers will do more to help young people build their knowledge by asking them questions than by giving them the answers.

Any question that helps students get on the right track or find the answers to their questions by themselves encourages them to participate in their own learning.

Encouraging Students to Use a Problem-Solving Approach at Every Stage in the Learning Process

Problem solving is an essential teaching and learning tool in several general education programs (e.g. pure sciences, social studies) and is an integral part any mathematical activity. Problem solving is not a separate theme, but rather a process that should be applied throughout the program and that provides a suitable context for learning concepts and acquiring skills.

> Problem solving is both a basic skill that students should develop and an effective teaching approach that promotes the development of mathematical knowledge, thinking skills, socio-affective attitudes and problem-solving strategies.[3]

Learning through problem-solving calls for the active involvement of the students and the use of questions. It is important that the teacher ask the students questions and that they in turn ask questions of each other and the teacher.

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2. Nadine Bednarz, "L'enseignement des mathématiques et le Québec de l'an 2000," excerpted from Richard Pallascio, ed., *Mathématiquement vôtre! Défis et perspectives pour l'enseignement des mathématiques* (Montréal: Les éditions Agence d'ARC inc., 1990), 69 (Free translation).

3. Québec, ministère de l'Éducation, *Mathematics Curriculum Guide, Elementary School, Booklet K, Problem Solving*, Code 16-2300-11A (Québec: ministère de l'Éducation, 1989), 47-51.

Some problem-solving tasks may be more difficult than others and the problems themselves can be quite varied. For instance, the students may encounter the following:

> ...problems with solutions requiring students to choose an appropriate combination of knowledge or skills from among several combinations seen in the past.[4]

They may even encounter the following:

> ... problems requiring students to create a new combination of knowledge and skills, exercise a great deal of intellectual independence and use plausible reasoning in order to solve them.[5]

Problem solving is a very effective means of developing knowledge and skills. The quality of learning depends on the variety of problems assigned and on their level of difficulty. In a learning context, the students can even be presented with very challenging problems. By solving these problems, the students can discover such things as properties, relationships and strategies by themselves. A wide variety of problems allows the students to conceptualize their knowledge and develop numerous problem-solving strategies. Problem solving is a way of learning and a way of teaching.

The problems can be related to the students' environment and used at various stages in the learning process. Problem solving can help students learn new concepts and develop skills or help them expand their knowledge and reinforce what they have learned.

Thus, problems provide an opportunity to:

- apply and integrate mathematical knowledge (e.g. concepts, properties, algorithms, techniques, procedures);
- develop intellectual skills (e.g. organizing, structuring, abstracting, analyzing, synthesizing, estimating, generalizing, deducing, justifying);
- develop positive attitudes (e.g. becoming aware of one's potential, respecting the opinions of others, and being imaginative and creative as well as rigorous and precise);
- use different problem-solving strategies (e.g. looking for patterns, representing a problem by means of a figure or a graph, constructing a table, referring to a known model, using a formula, formulating an equation, working backwards).

4. Québec, ministère de l'Éducation, *Mathematics Curriculum Guide, Elementary School, Booklet K, Problem Solving*, Code 16-2300-11A (Québec: ministère de l'Éducation, 1989), 15.

5. Ibid, 15.

The emphasis on problem solving does not mean that exercises have no part in the teaching or in the learning of mathematics. Exercises play a different role, but one that is complementary to that of problem solving. For instance, exercises can help students consolidate skills and habits that they have already begun to develop. They can also give students the opportunity to apply definitions and properties that they have already learned in class. Exercises can neither replace nor be replaced by problems.

By using a problem-solving approach, the students become accustomed to referring to a known mathematical model and are thereby more likely to attain the terminal objectives. The teacher should also assist the students in using a procedure that will enable them to acquire more knowledge and generate other models. This will help the students attain the global objectives in accordance with the first guiding principle, namely, to encourage the students' active participation.

Students must have the opportunity to analyze their work methods and organize their thinking. In short, they must be able to learn how to learn.

Encouraging Students to Use the Appropriate Technology for Each Task

> All industrialized countries have experienced a shift from an industrial to an information society, a shift that has transformed both the aspects of mathematics that need to be transmitted to students and the concepts and procedures they must master if they are to be self-fulfilled, productive citizens in the next century.
>
> [...] This social and economic shift can be attributed, at least in part, to the availability of low-cost calculators, computers, and other technology. The use of this technology has dramatically changed the nature of the physical, life and social sciences; business; industry; and government. The relatively slow mechanical means of communication—the voice and the printed page—have been supplemented by electronic communication, enabling information to be shared almost instantly with persons—or machines—anywhere. [...] The impact of this technological shift is no longer an intellectual abstraction. It has become an economic reality. Today, the pace of economic change is being accelerated by continued innovation in communications and computer technology.[6]
>
> [...] changes in technology and the broadening of the areas in which mathematics is applied have resulted in growth and changes in the discipline of mathematics itself. Davis and Hersh (1981) claim that we are now in a golden age of mathematical production, with more than half of all mathematics having been invented since World War II.[7]

Since technology has influenced mathematics and its applications, students must learn to use modern electronic tools like scientific calculators, graphing calculators, drawing software, and utility software (e.g. spreadsheet programs, word processing systems and database managers).

Technology does not guarantee that students will do well in mathematics, since calculators and computers, like a word processor for a writer, are merely tools. However, technology does enable students to understand and master new concepts more quickly.

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6. Thomas A. Romberg, ed. *Curriculum and Evaluation Standards for School Mathematics* (Reston, Va.: National Council of Teachers of Mathematics, 1989), 3.

7. Ibid, 7-8.

Connection with Previous Programs

With continuity in learning, students can review topics they have already studied and further develop their conceptions and representations. This mathematics program enables students to build on the knowledge acquired in elementary school and in the first three years of secondary school.

This learning process will be dynamic if the learning activities allow the students to use their previously acquired knowledge and skills in new situations and help them become more proficient at applying what they have learned.

As they acquire new knowledge, the students will review the following skills and concepts acquired in previous programs:

- number sense, facility with operations, proportionality, the concept of a variable and spatial relationships;
- the habit of estimating;
- the types of dependence characterizing the relationship between the variables in a situation;
- translation from one mode of representation to another;
- definitions, properties, theorems or corollaries related to different geometric concepts;
- the ability to organize and process statistical data;
- simulation of random events and the concept of probability.

Evaluation of Learning

Orientations and Practices Relating to the Evaluation of Learning

> The evaluation of student learning has come in for a great deal of discussion in the Québec education system over the last decade and it is surely no exaggeration to say that this field has been and to some extent remains a subject of scrutiny. Teachers today are more knowledgeable about the evaluation of student learning than they were in the past...[8].

It is important to draw on all the available expertise in evaluation and ensure that evaluation practices increasingly tie in with the essential learning pursued in the programs of study. Thus, the aim should be to establish greater consistency between the spirit of these programs and evaluation practices.

8. Conseil supérieur de l'éducation, *Évaluer les apprentissages au primaire : un équilibre à trouver* (Québec, Direction des communications du CSE), 1992, 1 (Free translation).

Procedures for Evaluating Learning

When evaluating student learning, teachers should keep in mind the purpose of evaluation. Whether the goal is to give immediate educational feedback (formative evaluation) or to determine whether one or more terminal objectives have been attained (summative evaluation), evaluation provides individual students with useful information about their learning progress. It also helps teachers to assess the organization of program content and the effectiveness of teaching methods. Since the program is aimed at helping students acquire a solid basic education and the skills that will enable them to adapt to a constantly changing society,

> ...the evaluation of learning should take into account the various components of human development and the complex nature of education, [and] be consistent with the learning activities carried out in the classroom...[9].

In this program, the students not only acquire knowledge, but also learn how to investigate, communicate, represent, reason and use a variety of approaches in order to solve problems. They also acquire other skills and attitudes.

Because the students' knowledge, skills and attitudes are constantly evolving, it is necessary to create situations which will yield information that, after criterion- or norm-referenced interpretation, is likely to provide a reliable indication of each student's or group's knowledge.

Since "paper-and-pencil" evaluation may not be appropriate to every aspect of this program, a certain amount of adaptation will be necessary. Depending on the specific goals and in keeping with a spirit of diversification, the following means of evaluation could be appropriate:

- Log
- Oral presentation of a solution or a mathematical subject
- Quiz
- Class discussion
- Group project
- Interview
- Comprehensive examination comprising a number of sections
- Evaluation during computer-assisted learning activities
- Observation checklist
- Self-evaluation
- and so on

The different types of evaluation must also take into account the variety of learning activities:

- Manipulation activity
- Communication activity (oral or written, individual or group)
- Estimation activity
- Activity using a calculator
- Activity using a computer
- and so on

When planning educational evaluation, it is important to vary the means of evaluation. However, this does not mean that only one type of evaluation (i.e. diagnostic, formative or summative) should be used. Choices must be made in this regard.

The evaluation of learning, be it formative or summative, is essentially aimed at improving both learning and teaching.

9. Conseil supérieur de l'éducation, *Évaluer les apprentissages au primaire : un équilibre à trouver* (Québec, Direction des communications du CSE, 1992), 1 (Free translation).

As Esther Paradis notes in *L'évaluation des apprentissages : valoriser sa mission pédagogique,* it is essentially a matter of rediscovering the educational merit of evaluation.[10]

Relative Importance of the General Objectives

The following table shows the relative importance of each general objective.

General Objectives	%
1. To help the students develop their ability to use algebra.	55
2. To help the students develop their ability to analyze geometric situations.	35
3. To help the students develop a more critical attitude towards a statistical study.	10

10. Esther Paradis, *L'évaluation des apprentissages : valoriser sa mission pédagogique* (Québec, Fédération des enseignantes et des enseignants de commissions scolaires, Centrale de l'enseignement du Québec, 1992), 26 (Free translation).

Program Content

Program Structure

This program is made up of global, general, terminal and intermediate objectives. These objectives should reflect the aims of mathematics education and the guiding principles mentioned previously.

Global Objectives

Objectives that summarize the role that mathematics plays in providing students with the basic education they need to integrate into our changing society. These global objectives remain the same throughout the five years of secondary school and form the nucleus around which the objectives for each level are structured.

General Objectives

Objectives that specify the context in which the global objectives will be pursued and that describe in general terms the *expected educational outcomes* associated with each program theme. General objectives can be broken down into a set of terminal objectives.

Terminal Objectives

Objectives that clarify the general objectives and describe the *anticipated results*. Each terminal objective is described in three paragraphs:

- The first paragraph indicates what the students have already learned.

- The second paragraph provides criteria for determining whether the students have attained the terminal objective.

- The third paragraph outlines activities that are consistent with the general objective, the global objectives and the guiding principles. In this way, it reflects the spirit of the program.

The terminal objective is attained when the students are able to establish a link between a situation and acquired knowledge. This ability is directly related to attainment of the terminal objective and not to attainment of each of the underlying intermediate objectives, a complex object of knowledge being more than the sum of its parts. Hence, the primary goal is to have the students achieve the terminal objectives of the program. The degree to which the terminal objectives of the program are attained is directly related to the appropriateness of the measurement instruments, which must take into account the scope of the intermediate objectives and the context outlined by the general objective and the global objectives.

Intermediate Objectives

Objectives that specify the scope of a terminal objective, intermediate objectives might also be described as "reference objectives." They are not intended as a series of steps to be completed one after the other. Such a process would give a very fragmented picture of teaching and learning. Rather, intermediate objectives are:

- aspects of a theme that have been chosen for the program;
- clarifications to ensure that the terminal objective is clearly understood;
- guidelines that indicate the connection between the terminal objective and student learning;
- prerequisites for attaining a terminal objective.

Program Objectives

Global Objectives

Establishing Connections

Increasing the students' ability to establish connections between the knowledge they are acquiring and the knowledge they already have in mathematics and other disciplines, and encouraging them to view their knowledge as a tool that can be useful to them in everyday life.

Communicating

Increasing the students' ability to grasp and transmit information and to express their thoughts clearly, using mathematical language.

Problem Solving

Increasing the students' ability to analyze the data associated with a problem and use appropriate strategies to arrive at a solution that they will be able to verify, interpret and generalize.

Reasoning

Increasing the students' ability to formulate hypotheses and verify them using an inductive or a deductive method.

GENERAL OBJECTIVE 1

To help the students develop their ability to use algebra

Since we are now living in the information age, students should be equipped to handle, process and interpret the information they will encounter.

In Secondary II, the students learned that algebra was a powerful and useful language or communication tool. They were introduced to different modes of representation (e.g. numerical expressions, images or drawings, tables of values, graphs or diagrams, algebraic expressions, equations, formulas) which highlighted certain aspects of problems they had to solve.

In Secondary III, the students used algebra to derive general rules from a number of specific situations. Conversely, they applied general rules to individual cases. The students discovered the type of dependence characterizing the relationship between certain variables, especially the dependence represented by the graph of a straight line. They also continued learning about algebraic manipulations.

In Mathematics 436, the students build on this knowledge in a more formal way. They will analyze different ways of representing functions. More specifically, they will determine the properties of a function given its Cartesian coordinate graph. In addition, given the rule of correspondence of a function, the students will analyze the relationships between changes in the parameters of that rule and changes in the equivalent Cartesian coordinate graph. To save time and help the students better understand this material, it would be worthwhile to use a graphing calculator or a computer when exploring these topics in class.

Like any other language, algebra has its own rules and syntax, and it is important that the students observe them. The students will continue developing their ability to perform algebraic operations, exploring those that involve the laws of exponents, radicals, operations on algebraic expressions, factoring and systems of first- or second-degree equations in two variables. The students will thus have several effective problem-solving tools. They will then analyze polynomial functions of a degree less than three, using both the rules of correspondence and the graphs of these functions. Here again, a graphing calculator or a computer would be very useful and effective.

Analytic geometry illustrates the relationship between algebra and geometry. The students will begin by studying straight lines in the Cartesian plane and their equations in various forms. In the process, they will define the concepts of distance and slope and then use them to prove geometric propositions.

The formal approach to mathematics in this course should result in the use of set notation and logical symbols and connectives, which will enable the students to express mathematical ideas more precisely and concisely.[1] The teacher should therefore present and explain this notation and symbolism as the need arises and encourage the students to use it often. With practice, the students will find it easy to understand and apply.

1. See Québec, ministère de l'Éducation, *Information Document, Graphs, Notation and Symbols Used in Secondary Mathematics,* Code 16-3306A, May 1982. (Québec: ministère de l'Éducation, 1982).

Terminal Objective 1.1

To analyze situations involving functions, using different modes of representation

In Secondary II, the students had to use different modes of representation to describe and represent a situation. They learned to translate a situation into a first-degree equation. In studying ratios and proportions, they explored situations involving direct variation. In Secondary III, they studied situations in which the variables are directly or inversely proportional or in which one of the variables is proportional to the square of the other. In particular, they analyzed situations in which the relationship between the variables is linear (i.e. situations involving direct or partial variation). They were able to do this with a minimum of complex symbolism.

Students who have attained Terminal Objective 1.1 of this program will be able to use different modes of representation to analyze situations involving functions in a more formal way. After defining a function as the relationship between one independent real-number variable and one dependent real-number variable, the students will learn to write this symbolically as $y = f(x)$. They will learn to state the properties of a function given its Cartesian coordinate graph. They will also learn to associate certain changes in the parameters of the rule of correspondence of a function with changes in the equivalent Cartesian coordinate graph. The students can explore a variety of situations involving different types of functions (e.g. polynomial, inverse variation, rational, square root, step and exponential functions). However, the students will not have to distinguish between these types of functions or classify the situations from which they are derived. This part of the course provides an introduction to different types of functions, focusing on the concept of a function and its various modes of representation. The terminology used should be formally defined. In addition, it is important that the students examine many different situations. The shaded boxes in the following table indicate the scope of Terminal Objective 1.1.

TRANSLATIONS FROM ONE MODE OF REPRESENTATION TO ANOTHER

from \ to	words or drawing	table of values	graph	rule or equation
words or drawing		(shaded)	(shaded)	(shaded)
table of values			(shaded)	(shaded)
graph		(shaded)		(shaded)
rule or equation		(shaded)	(shaded)	

The use of a wide variety of situations giving rise to discussions and questions that involve analyzing functions is consistent with the global objectives, General Objective 1 and the guiding principles. The students will develop their powers of observation and their ability to analyze and synthesize a situation. They will learn to interpret graphs and understand the relationships between symbolic, graphic and numerical representations of the same situation. It may be useful (even necessary) to employ different learning aids and methods (i.e. "pencil-and-paper" exercises, graphing calculators and computers).

1.1

Intermediate Objectives

- To use symbols to represent a situation involving a function, indicating a source set, a target set and a rule of correspondence.
- To draw the Cartesian coordinate graph representing a situation involving a function, given an equivalent verbal description, table of values or rule of correspondence.
- To prepare the table of values for a situation involving a function, given an equivalent verbal description, rule of correspondence or Cartesian coordinate graph.
- To describe the properties of a Cartesian coordinate graph representing a function (these properties are listed below).

  - increasing or decreasing function
  - sign
  - rate of change
  - axes of symmetry, if any
  - maxima or minima, if any
  - x-intercept(s) (zeros)
  - y-intercept
  - domain and range

- To determine the relationships between changes in the parameters of the rule of correspondence of a function and changes in the equivalent Cartesian coordinate graph.

Terminal Objective 1.2

To transform an algebraic expression into an equivalent expression

In Secondary I, the students developed their understanding of the four operations on rational numbers as well as their ability to perform these operations. In Secondary II, they began to perform operations on certain expressions containing a variable. In Secondary III, they were able to perform operations on expressions containing exponents and to apply certain laws pertaining to positive integral exponents. They also performed operations on polynomials (added and subtracted polynomials, multiplied a monomial by a polynomial and a binomial by a binomial, and divided a polynomial by a monomial).

Students who have attained Terminal Objective 1.2 will be able to do the calculations involved in converting algebraic expressions into equivalent expressions. When transforming algebraic expressions, they should be able to apply the definitions and properties of rational exponents. The theory of radicals should not be emphasized. Exercises should simply involve operations on square roots as well as the rationalization of numerators and denominators (the numerators and denominators should consist of no more than binomials). The students should be able to perform standard operations on algebraic expressions such as simple rational expressions. Nevertheless, operations on polynomials will be emphasized. Divisions will be limited to those that involve finding the quotient of a polynomial and a binomial. Conversely, students should be able to factor polynomials by removing the common factor, by grouping, by finding a difference of squares or a second-degree trinomial with integral coefficients or by completing a square. A geometric approach (e.g. algebra tiles) may be useful for introducing factoring and will make these techniques more concrete. Lastly, bear in mind that the students must be able to apply these techniques regularly in order to master them. It is therefore important to give the students the opportunity to use these methodsas often as possible.

The shaded boxes in the following table indicate the scope of Terminal Objective 1.2.

TRANSLATIONS FROM ONE MODE OF REPRESENTATION TO ANOTHER

to from	words or drawing	table of values	graph	rule or equation
words or drawing				
table of values				
graph				
rule or equation				(shaded)

Activities in which the students gradually develop their knowledge of the structure of algebra, their understanding of the laws of algebra and their ability to apply its techniques are consistent with the global objectives, General Objective 1 and the guiding principles. While ensuring that the students understand these methods, the teacher should also help them develop certain habits. The students will have many opportunities to use the skills they have already acquired or developed.

1.2

Intermediate Objectives

- To apply the theory of exponents in transforming algebraic expressions.
- To perform operations (addition, subtraction, multiplication division and exponentiation) on algebraic expressions and on polynomials in particular.
- To factor a given polynomial.
- To transform rational algebraic expressions by dividing or factoring them.

Terminal Objective 1.3

To analyze polynomial functions of a degree less than 3

In Secondary II, the students began to study algebra and used various modes of representation to describe and represent a situation. In Secondary III, they analyzed various situations involving relationships between two variables. In studying the material covered in Terminal Objective 1.1. of this course, the students analyzed several situations involving functions (more specifically, they formally defined the concept of a function, the properties of functions given the equivalent Cartesian coordinate graphs, and the terminology used). They also studied the relationships between changes in the parameters of the rule of correspondence of a function and changes in the equivalent Cartesian coordinate graph.

Students who have attained Terminal Objective 1.3 of this program will be able to analyze real polynomial functions, focusing on the relationships between the various forms of the rule of correspondence of a function and the equivalent Cartesian coordinate graph. The students will learn to use a graphing calculator or a computer to determine the coordinates of important points on a graph with the required degree of precision. They will also study formulas for finding the zero(s) of polynomial functions (the roots of the corresponding equations) and their extremes if any. Students should be able to transform the rule of correspondence of a function, draw the Cartesian coordinate graph of a function, give its main characteristics and determine the rule of correspondence of a function if its Cartesian coordinate graph is given or described. This type of in-depth analysis will be limited to polynomial functions of degree 0, 1 or 2. Furthermore, the sum, difference and product of two polynomial functions will be examined. The students could create their own functions and then graph them. They can then observe the properties of the resulting Cartesian coordinate graphs, comparing and describing them. The student should explore and observe many of these functions, using the appropriate terminology to describe the resulting graphs without trying to formalize what they have learned. A graphing calculator or a computer would be very useful to help the students attain Terminal Objective 1.3.

Activities that bring about discussions and questions and in which the students will develop their powers of observation and their ability to analyze and synthesize situations are consistent with the global objectives, General Objective 1 and the guiding principles. The students will learn to classify a polynomial function by its rule of correspondence and Cartesian coordinate graph and to understand the relationships between these two modes of representation. Graphing calculators and computers are efficient teaching tools for this topic because they make it possible to analyze a greater number of functions, thereby facilitating the task of synthesizing this material. In addition, students will be able to use their knowledge and skills to solve a variety of problems based on mathematical, real, realistic and/or imaginary situations.

1.3

Intermediate Objectives

- To draw the Cartesian coordinate graph (a straight line) of a real polynomial function of degree 0 or 1, given the equivalent rule of correspondence.

- To determine, from its rule of correspondence, the following information about a real polynomial function of degree 0 or 1: its rate of change, its *x*-intercept (zero), its *y*-intercept, its domain and range, its sign, whether it is constant, increasing or decreasing, and the member of its domain associated with a given image.

- To draw the Cartesian coordinate graph (a parabola) of a real polynomial function of degree 2, given the equivalent rule of correspondence.

- To determine, from its rule of correspondence, the following information about a real polynomial function of degree 2: its extreme (vertex of the parabola), its zeros (if any), the sum and product of the zeros, its *y*-intercept, its domain and range, the intervals within which it is increasing and decreasing, its sign, and the member(s) of its domain associated with a given image.

- To use algebra to convert the rule of correspondence for a real polynomial function of degree 2 from the general form

  $$f(x) = ax^2 + bx + c, a \neq o$$

  into the standard form

  $$f(x) = a(x - h)^2 + k, a \neq o$$

  and vice versa.

- To determine the relationships between changes in the parameters of the rule of correspondence for a real polynomial function of a degree less than 3 and changes in the equivalent Cartesian coordinate graph.

- To determine the rule of correspondence of a real polynomial function of degree 0 or 1 represented by a straight line, given the slope of that line and a point on that line or given two points on that line.

- To determine the rule of correspondence of a real polynomial function of degree 2 represented by a parabola, given the vertex of that parabola and another point on that parabola or given its zeros and another point.

- To graph the sum, difference and product of two real polynomial functions, given the graph or the rule of correspondence of each of these functions.

Terminal Objective 1.4

To solve problems using a system of equations in two variables

In Secondary II, the students acquired certain skills that enabled them to represent a situation by a first-degree equation and then solve it. In Secondary III, they continued developing their ability to perform operations on algebraic expressions. In studying the material covered in Terminal Objectives 1.1 and 1.3 of this course, they used algebra to analyze functions. In exploring the material covered in Terminal Objective 1.2 of this course, they became familiar with new algebraic techniques and developed their ability to use them.

Students who have attained Terminal Objective 1.4 of this program will be able to represent a situation by a system of equations and solve them algebraically or by graphing. A graphing calculator or a computer can be used to solve systems of equations graphically when the equations must first be transformed into rules of correspondence. The students will study several algebraic methods of solving a system of two first-degree equations in two variables, but they will be free to use the methods they prefer. The substitution method should be emphasized for systems consisting of a first-degree equation in two variables and a second-degree equation in two variables. In the Cartesian plane, the solution of this system will correspond to the intersection of the two graphs representing these two equations (i.e. an empty set, a singleton or a pair). In all cases, the students should be able to solve problems arising from mathematical, real, realistic and/or imaginary situations.

A wide variety of situations in which students must analyze the relationships between the data in the problem, formulate a system of equations, solve it graphically or algebraically and interpret the results is consistent with the global objectives, General Objective 1 and the guiding principles. The students can also be asked to solve a system of equations unrelated to a situation. It would be useful to employ different learning aids and methods (i.e. "pencil-and-paper" exercises, graphing calculators and computers).

1.4

Intermediate Objectives

- To represent a situation by a system of two first-degree equations in two variables.
- To solve a system of two first-degree equations in two variables by graphing it.
- To solve a system of two first-degree equations in two variables algebraically.
- To represent a situation by a system of two equations, one being of the first degree in two variables and the other being of the second degree in two variables.
- To use a graph to solve a system of two equations, one being of the first degree in two variables and the other being of the second degree in two variables.
- To use algebra to solve a system of two equations, one being of the first degree in two variables and the other being of the second degree in two variables.

Terminal Objective 1.5

To solve problems in analytic geometry

Since Secondary II, the students have been acquiring algebraic knowledge and skills (i.e. first-degree equations in one variable, operations on polynomials, the relationship between variables in a situation, functions, transformations of algebraic expressions, and systems of equations). In exploring the material covered in Terminal Objectives 1.1 to 1.4 of this course, they learned to use advanced algebraic methods. The students have also been studying geometry since Secondary I. [2]

Students who have attained Terminal Objective 1.5 of this program will be able to use their algebraic and geometric knowledge and skills to solve problems in the Cartesian plane in a more formal way. They will begin by using analytic geometry to study straight lines. This will involve drawing straight lines, examining the role of parameters, and finding different forms of equations, the distance between two points (absolute value must be defined), the distance between a point and a straight line, the point of division of a segment as well as the area and perimeter of polygons. The students will have to show and justify all the steps in their solutions. They will then go on to prove geometric propositions formally (with the teacher at first, then in teams and eventually by themselves). Appendix 2 (page 43) gives a list of simple propositions that are useful and easy to deal with. When introducing the students to the task of presenting a proof, it is important to guide them through the process by helping them acquire skills that will be developed later. Proofs should not be emphasized in summative evaluation. However, the students should be able to apply these propositions in specific cases.

Activities that will help the students gradually expand their knowledge of analytic geometry, while developing their ability to justify and prove propositions are consistent with the global objectives, General Objective 1 and the guiding principles. The students should focus on understanding proofs rather than on the mechanics of presenting them. They will be able to use their knowledge and skills to solve a variety of problems. They will also discover the effectiveness of methods that show the relationship between algebra and geometry.

2. See Appendix 1, page 41.

1.5

Intermediate Objectives

- To determine the slope of a straight line that passes through two given points.
- To determine the slope, x-intercept and y-intercept of a straight line from a given equation.
- To draw a straight line in a Cartesian plane, given the slope of the line and a point on the line.
- To determine the equation of a straight line, given any of the following combinations: its slope and a point on the line, two points on the line, the x-intercept and y-intercept, or a point on the line and the equation of a parallel or perpendicular line.
- To transform the equation of a straight line algebraically.
- To determine if two straight lines intersect, or if they are perpendicular, parallel and distinct, or parallel and coincident by comparing their parameters and equations.
- To determine the distance between two points or between a point and a straight line.
- To determine the coordinates of the point of division of a segment, given the coordinates of its endpoints and other relevant data.
- To determine the area and the perimeter of polygons, given the coordinates of the vertices.
- To prove propositions using analytic geometry.[3]

3. See Appendix 2, page 43.

GENERAL OBJECTIVE 2

To help the students develop their ability to analyze geometric situations

One of the major reasons for teaching geometry "is to build the kind of strong geometric intuition that has been shown to be an important factor for success on the job and in college." [4]

The students gradually develop their geometric thinking skills. They first learn to recognize shapes and then analyze the different properties of these shapes before establishing relationships between the properties and making simple deductions. Through numerous activities involving active exploration and observation, the students create a system of relationships pertaining to triangles, quadrilaterals, circles, regular polygons, isometric transformations, dilatations and solids. [5] In Mathematics 436, this system will be expanded to include the concepts of isometry, similarity, and equivalence as well as trigonometric ratios.

When formally defining the concepts of "isometry" and "similarity transformation" and studying them in greater detail, the students will apply their knowledge of geometric transformations. In the process, they will discover the minimum conditions required for triangles to be isometric or similar (students can discover this through experimentation and observation) as well as the properties of isometric or similar plane figures. [6] Here the students can once again apply their knowledge of ratios and proportions. Equivalent plane figures will be defined as "figures with the same area" and their main properties will be studied.

In Mathematics 436, the students must continue honing the spatial sense they began developing in Secondary III. This skill, which makes it possible to create and manipulate images of objects, will be used to study isometric and similar right solids. In the process, the geometric transformations studied up to now will be very informally examined in three-dimensional space. The students will study and use the properties of these solids. Equivalent solids will be defined as "solids with the same volume." The students will then establish relationships between the total surface area and the volume of different solids. [7]

Other activities will help the students discover new techniques for solving problems related to triangles (i.e. sine, cosine and tangent ratios in right triangles; the law of sines and the law of cosines in any triangle). Naturally, all these definitions, properties and laws will be used to solve geometric problems involving two- or three-dimensional figures. As a rule, the students should justify any statement or problem-solving procedure. The geometry in this course should be logical and well thought-out and should prepare the students for the formal proofs they will encounter later on.

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4. Arthur F. Coxford, et al., "Geometry from Multiple Perspectives," in *Curriculum and Evaluation Standards for School Mathematics, Addenda Series Grades 9 to 12* (Reston, Va.: National Council of Teachers of Mathematics, 1991), page v.

5. See Appendix 1, page 41.

6. See Appendix 3, page 43.

7. See Appendix 4, page 47.

"Students need to have many informal experiences that involve reasoning and arguing to support conjectures before they are likely to understand the need for, or the value of, a formal proof." [8]

As a result, students should be given "more chances to investigate and think about geometric questions on their own or in small groups."[9] Modern technology can be very useful at this point, since some computer programs can be used to explore geometric problems, thereby enabling students to formulate conjectures, discuss them and test them without outside help.

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8. Arthur F. Coxford, et al., "Geometry from Multiple Perspectives," in *Curriculum and Evaluation Standards for School Mathematics, Addenda Series Grades 9 to 12* (Reston, Va.: National Council of Teachers of Mathematics, 1991), 51.

9. Ibid, 64.

Terminal Objective 2.1

To solve problems using the concepts of isometry, similarity and equivalence

Since Secondary I, the students have been constructing plane figures using isometric transformations, dilatations and their composites. They have also explored the properties of these transformations. They have made these constructions using geometry instruments as well as a rule for transforming the coordinates of points in a Cartesian plane (reflection about the axes or the bisectors of the quadrants, rotations centred at the origin and for which the rotation angle is a multiple of 90°, and dilatations centred at the origin). The students have studied the concept of an inverse transformation and identified the one transformation equivalent to a composite of transformations. In addition, they have developed their spatial sense and increased their knowledge of solids (they have created, represented, classified, constructed and analyzed them and split them into sections). They have also determined the measurements of these solids.

Students who have attained Terminal Objective 2.1 of this program will have increased their knowledge of isometric, similar and equivalent two- or three-dimensional figures so that they can solve different problems. First, the students will describe isometries or similarity transformations and then try to identify the isometry or similarity transformation (or the composite of two geometric transformations) that maps one isometric or similar plane figure onto another. They will then state the main properties of isometric, similar or equivalent plane figures and use them to solve problems. Lastly, they will apply these concepts to solids that are isometric, similar, equivalent or equal in total surface area. For both solids and polygons, the students will solve problems by organizing their solutions, justifying the steps in their reasoning and referring to definitions, theorems and properties they have already studied. The students can gradually learn how to present formal proofs by endeavouring to provide structured solutions involving sound, systematic argumentation.

Activities in which the students will have to identify properties or theorems, prove them and then use them to solve problems are consistent with the global objectives, General Objective 2 and the guiding principles. The students will learn to distinguish a conjecture from a certainty and a hypothesis from a conclusion. By always making an effort to explain their logic when analyzing a geometric situation or solving a problem, they will gradually learn to use the formal reasoning required to present proofs.

2.1

Intermediate Objectives

- To define isometries and similarity transformations by means of geometric transformations and their composites.
- To accurately describe the geometric transformation or the simplest composite of geometric transformations that maps one isometric or similar plane figure onto another, given two isometric or similar plane figures.
- To characterize isometric, similar or equivalent plane figures.
- To determine the properties (e.g. measures of angles and sides, perimeters, areas) of isometric, similar or equivalent plane figures.
- To state the minimum conditions required for two triangles to be isometric or similar.
- To characterize solids that are isometric, similar, equivalent or equal in total surface area.
- To determine the properties (e.g. measures of angles and sides, perimeters, areas, volumes) of solids that are isometric, similar, equivalent, or equal in surface area.
- To determine certain measurements of similar right solids or spheres, given other measurements of these figures, a ratio (of lengths, of surface areas or of volumes), or data that can be used to find this ratio.
- To justify an assertion used in solving a problem.[10]

10. See Appendix 3, page 45.

Terminal Objective 2.2

To solve problems using trigonometric ratios

In the first cycle of secondary school, the students learned the concepts of ratio and proportion. In studying the material covered in Terminal Objective 2.1 of this course, they learned about similar figures and developed their ability to reason more systematically.

Students who have attained Terminal Objective 2.2 of this program will be able to use trigonometric ratios, the law of sines and the law of cosines to solve problems that involve determining measurements in a triangle. Exercises that simply involve finding the measure of a side or an angle in a triangle will not suffice. Instead, the students should be required to use these measurements to solve problems.

Activities in which the students can discover that trigonometric ratios are derived from ratios of similitude in similar right triangles are consistent with the global objectives, General Objective 2 and the guiding principles. The students will work out the law of sines and the law of cosines and use these tools to solve a variety of problems that involve determining the measurements in triangles (e.g. finding measurements that cannot be determined directly, locating a position, land surveying). Here too, the students will gradually learn to use formal reasoning by always making an effort to justify each step in their solutions.

2.2

Intermediate Objectives

- To calculate the measure of a side or an angle in a right triangle, given relevant data and using a trigonometric ratio.
- To calculate the measure of a side or an angle in a triangle, given relevant data and using the law of sines or the law of cosines.
- To justify an assertion used in solving a problem.[11]

11. See Appendix 3, page 45.

GENERAL OBJECTIVE 3

To help the students develop a more critical attitude towards a statistical study

To be informed and productive, a person must have a certain ability to handle data and make intelligent decisions based on quantitative and qualitative arguments. In addition to knowing how to find the right numerical answer, he or she must be able to analyze situations critically. The students will learn how to ask pertinent questions and present an analysis while developing their critical sense.

In the first cycle of secondary school, the students organized and presented data in tables and graphs. They also saw that they could use certain descriptive measures (mean, median, mode, range) to synthesize data and thus provide information on various phenomena. Statistical analysis calls for other data besides measures of central tendency. In Secondary IV, the students will therefore begin examining measures of position and will be introduced to the concept of dispersion.

The students should also be encouraged to analyze the way in which data is gathered from various sources and to assess the strengths and weaknesses of the data-gathering process. To do this, the students will have to learn certain methods of analyzing data.

With this approach, the students will learn to use data rather than produce it. They should be given the opportunity to investigate and discuss such things as public opinion polls, media ratings and census data.

Terminal Objective 3.1

To solve problems using measures of position

In the first cycle of secondary school, the students learned to use certain tools (measures of central tendency and range) to analyze information and presented data in the form of tables or graphs (bar, broken-line and circle graphs as well as histograms).

Students who have attained Terminal Objective 3.1 of this program will be able to solve problems using the graphic and numerical tools they have to analyze information. They are to use these tools to study the variability of a distribution. The students will use measures of position to determine the rank of a data value in relation to other values in a distribution, or to identify the possible variations among the data values in the distribution. In continuing to explore methods of analyzing data, the students will increase their knowledge of mathematical models by studying the box-and-whisker plot. This graph not only highlights certain characteristics of a distribution, but also gives the students an idea about the dispersion of the data.

Activities in which the students can present information about a set of data using a graph or certain measures describing that distribution are consistent with the global objectives, General Objective 3, and the guiding principles. Technology should be used to facilitate the analysis and interpretation of a situation. The analysis and presentation of the situation should be emphasized. In this way, the students will learn to interpret graphs and understand the connections between graphic and numerical representations of the same situation.

3.1

Intermediate Objectives

- To distinguish between measures of position, measures of central tendency and measures of dispersion.
- To assign a quintile or a percentile rank to a data value, if necessary.
- To determine the data value(s) that are assigned a quintile rank or a percentile rank.
- To use measures of position to compare data.
- To construct a box-and-whisker plot.
- To interpret a box-and-whisker plot.
- To find qualitative information about the dispersion of the data in a one-variable distribution, using measures of position and measures of central tendency.

Terminal Objective 3.2

To solve problems that involve gathering data

In the first cycle of secondary school, the students organized data in the form of tables or graphs, usually working with given information. They continued to study phenomena involving chance and also used certain measures to describe this data (mean, median, mode, range).

Students who have attained Terminal Objective 3.2 of this program will be able to assess the reliability of the sample and the relevance of the data used in making predictions about a given population. To determine if the data is relevant, one must ascertain whether or not it is representative. If the initial hypothesis is appropriate, the sample should provide an accurate picture of the population under study. The students should check the size of the sample and the data-gathering methods to ensure that a study is as unbiased and error-free as possible. The students already know several ways of describing data graphically or numerically. They must learn to follow certain principles in processing data to ensure that they draw appropriate conclusions.

Activities in which the students will develop a critical attitude towards survey data are consistent with the global objectives, General Objective 3 and the guiding principles. The students should become aware that a survey consists of several elements, all capable of affecting the accuracy of the results. During their discussions and investigations, the students should watch for biases in the selection of data, for errors in measurement and for distortions in graphic or numerical representations of data, both in the media and in their own work.

3.2

Intermediate Objectives

- To distinguish between a sample and a population.
- To justify the decision to prepare a census, a poll or a study to obtain information.
- To describe the characteristics of a representative sample of a given population.
- To choose an appropriate sampling method when gathering information.
- To determine the possible sources of bias during the data-gathering process.
- To compare two samples from the same population.

Appendices

Appendix 1 Principles of Geometry Studied in the First Cycle of Secondary School

Expanding their knowledge gradually, the students have studied properties of two- and three-dimensional figures as well as the properties of geometric transformations. These properties are summarized below and should be included with those to be introduced in *Mathematics 436*. Since the students started using the term "isometric transformation" in Secondary III, statements related to the concept of congruence (2, 7, 13, 14, 16 and 23) have been changed. In these cases, the concept of congruence has been replaced by the concept of isometry, which is a broader term. The principles given below can be used to determine measurements in certain figures and to justify certain steps involved in solving problems.

Note: When the segments, sides or angles involved are equal in measure, they can be described as *congruent.*

1. Adjacent angles whose external sides are in a straight line are supplementary.
2. Vertically opposite angles are isometric.
3. The sum of the measures of the interior angles of a triangle is 180^{o}.
4. In any triangle, the length of any side is less than the sum of the lengths of the other two sides.
5. In any triangle, the length of any side is greater than the difference of the lengths of the other two sides.
6. In any triangle, the longest side is opposite the largest angle.
7. In any isosceles triangle, the angles opposite the isometric sides are isometric.
8. In any equilateral triangle, each of the angles measures 60^{o}.
9. In any right triangle, the acute angles are complementary.
10. In any isosceles right triangle, each of the acute angles measures 45^{o}.
11. The axis of symmetry of an isosceles triangle contains a median, a perpendicular bisector, an angle bisector and an altitude of the triangle.
12. The axes of symmetry of an equilateral triangle contain the medians, perpendicular bisectors, angle bisectors and altitudes of the triangle.
13. The opposite angles of a parallelogram are isometric.
14. The opposite sides of a parallelogram are isometric.
15. The diagonals of a parallelogram bisect each other.
16. The diagonals of a rectangle are isometric.
17. The diagonals of a rhombus are perpendicular to each other.
18. The diagonals from one vertex of a convex polygon form n - 2 triangles, where n is the number of sides in that polygon.
19. In a convex polygon, the sum of the measures of the exterior angles, one at each vertex is 360^{o}.

Appendix 1 (cont'd.)

20. The sum of the measures of the interior angles of a polygon is 180^{o} (n - 2) where n is the number of sides in the polygon.

21. Three non-collinear points determine one and only one circle.

22. All the perpendicular bisectors of the chords of a circle meet at the centre of the circle.

23. All the diameters of a circle are isometric.

24. In a circle, the measure of the radius is half the measure of the diameter.

25. The axes of symmetry of a circle contain its centre.

26. The ratio of the circumference of a circle to its diameter is a constant known as π.

27. In a circle, the measure of the central angle is equal to the measure of its intercepted arc.

28. In a circle, the ratio of the measures of two central angles is equal to the ratio of the measures of their intercepted arcs.

29. The area of a circle is equal to πr^2.

30. In a circle, the ratio of the areas of two sectors is equal to the ratio of the measures of their central angles.

31. In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other sides.

32. A triangle is right-angled if the square of the length of one of its sides is equal to the sum of the squares of the lengths of the other two sides.

33. In any convex polyhedron, the sum of the number of vertices and the number of faces is equal to the number of edges plus two.

34. An isometry preserves collinearity, parallelism, the order of points, distances and measures of angles. In addition, translations and rotations preserve the orientation of the plane.

35. Any dilatation preserves collinearity, parallelism, the order of points, the orientation of the plane, the measures of angles and the ratio of the distances.

36. Any translation and any dilatation will transform a straight line into another line parallel to it.

Appendix 2 Deductive Reasoning in Analytic Geometry

The students are assumed to have the following knowledge and skills:

- The formula for finding the distance between two points (based on the Pythagorean theorem)
- The formula for calculating the distance between a point and a straight line
- The formula or a method for finding the coordinates of the point of division of a segment
- The general form of the equation of a straight line
- The functional form of the equation of a straight line (slope-intercept)
- The symmetric form of the equation of a straight line
- The role of the parameters in the various forms of the equation of a straight line (general, functional and standard forms)

The following propositions are considered to be true:

- The x- and y-axes are orthogonal.
- Two straight lines that are not parallel to the y-axis are parallel if and only if their slopes are equal.
- Two straight lines that are not parallel to the y-axis are perpendicular if and only if their slopes are negative reciprocals.
- A system of axes can always be arranged so that two consecutive vertices of a given polygon are on the x-axis, one of these vertices being located at the origin.

Appendix 2 (cont'd.)

The students can prove the following propositions using the information on the previous page.

1. The segment joining the midpoints of two sides of a triangle is parallel to the third side and its length is one-half the length of the third side.

2. The segment joining the midpoints of the non-parallel sides of a trapezoid is parallel to the bases and its length is one-half the sum of the lengths of the bases.

3. The segments joining the midpoints of the opposite sides of a quadrilateral and the segment joining the midpoints of the diagonals are concurrent in a point that is the midpoint of each of these segments.

4. A segment connecting a vertex of a parallelogram to the midpoint of one of the non-adjacent sides intersects the opposite diagonal at a point that divides both that segment and the diagonal in the ratio of 1 : 2.

5. The midpoint of the hypotenuse of a right triangle is equidistant from the three vertices.

6. The midpoints of the sides of any quadrilateral are the vertices of a parallelogram.

7. The three perpendicular bisectors of the sides of a triangle are concurrent in a point that is equidistant from the three vertices.

8. The three medians of a triangle are concurrent and trisect one another at the point of concurrency.

9. In any triangle, if a is the length of a side opposite an acute angle, if b and c are the lengths of the other two sides and if AH is the length of the projection of side c onto side b, then the following relationship is true:

$$a^2 = b^2 + c^2 - 2\,b(\text{AH})$$

10. In any triangle, the sum of the squares of the lengths of the medians is equal to three-quarters of the sum of the squares of the lengths of the sides.

11. If ABCD is a parallelogram and if E is the midpoint of side AD, F is the midpoint of side AB, G is the midpoint of side BC and H is the midpoint of side CD, then the segments AH, FC, BE, and DG intersect to form another parallelogram.

12. The sum of the squares of the distances between a given point and two opposite vertices of a rectangle is equal to the sum of the squares of the distances between this point and the other two vertices of the rectangle.

Of course, other geometric propositions can be proven.

Appendix 3 Principles of Geometry Introduced in *Mathematics 436*

1. If a transversal intersects two parallel lines then:
   - the alternate interior angles are isometric;
   - the alternate exterior angles are isometric;
   - the corresponding angles are isometric.

2. If two corresponding (or alternate interior or alternate exterior) angles are isometric, then they are formed by two parallel lines and a transversal.

3. The corresponding parts of isometric plane figures or solids are equal in measure.

4. Plane figures or solids are isometric if and only if there is an isometry that maps one figure onto the other.

5. If the corresponding sides of two triangles are isometric, then the triangles are isometric.

6. If two sides and the contained angle of one triangle and the corresponding two sides and contained angle of another triangle are isometric, then the triangles are isometric.

7. If two angles and a side of one triangle and two angles and the corresponding side of another triangle are isometric, then the triangles are isometric.

8. Transversals intersected by parallel lines are divided into segments of proportional lengths.

9. The line segment joining the midpoints of two sides of a triangle is parallel to the third side and its length is one-half the length of the third side.

10. Any straight line that intersects two sides of a triangle and is parallel to the third side forms a smaller triangle similar to the larger triangle.

11. Plane figures or solids are similar if and only if there is a similarity transformation that maps one figure onto the other.

12. If two angles of one triangle and the two angles of another triangle are isometric, then the triangles are similar.

13. If the lengths of the corresponding sides of two triangles are in proportion, then the triangles are similar.

14. If the lengths of two sides of one triangle are proportional to the lengths of two sides of another triangle and the contained angles are isometric, then the triangles are similar.

15. In similar plane figures or solids:

    - the ratio between the measures of the corresponding angles is 1;
    - the ratio between the lengths of the corresponding segments is equal to the ratio between the lengths of the corresponding sides;
    - the ratio of the areas is equal to the square of the ratio between the lengths of the corresponding sides;
    - the ratio of the volumes is equal to the cube of the ratio between the lengths of the corresponding sides.

Appendix 3 (cont'd.)

16. Plane figures or solids with a scale factor of 1 are isometric.

17. In a right triangle, the length of the side opposite a 30° angle is equal to half the length of the hypotenuse.

18. The law of sines:

    The lengths of the sides of any triangle are proportional to the sines of the angles opposite these sides.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

19. The law of cosines:

    The square of the length of a side of any triangle is equal to the sum of the squares of the lengths of the other two sides minus twice the product of the lengths of the other two sides multiplied by the cosine of the contained angle.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = c^2 + a^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Appendix 4 Properties of Two- and Three-Dimensional Figures

1. Regular polygons have the smallest perimeter of all equivalent polygons with *n* sides.

2. Of two equivalent convex polygons, the polygon with the most sides will have the smaller perimeter. (As a limiting case, an equivalent circle will have the smaller perimeter.)

3. Cubes have the largest volume of all rectangular prisms with the same total surface area.

4. Spheres have the largest volume of all solids with the same total surface area.

5. Cubes have the smallest total surface area of all rectangular prisms with the same volume.

6. Spheres have the smallest total surface area of all solids with the same volume.

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