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in the presence of optional
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GERAD HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

Tél. : 514 340-6053
Télec. : 514 340-5665
info@gerad.ca
www.gerad.ca

Pricing in marketing channels in the presence of optional contingent products

Peter M. Kort^a

Sihem Taboubi^b

Georges Zaccour^c

^a *Department of Econometrics and Operations Research,
Tilburg University, 5000 LE Tilburg, The Netherlands*

^b *GERAD and Marketing Department, HEC Montréal,
Montréal, Québec, Canada*

^c *GERAD and Department of Decision Sciences, HEC
Montréal, Montréal, Québec, Canada*

kort@tilburguniversity.edu

sihem.taboubi@hec.ca

georges.zaccour@hec.ca

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Abstract: The technological developments observed in the last two decades contributed to the digitalization of products and the introduction of devices to consume this digital content. Many online retailers launched their own devices, which had an effect on the pricing strategies of the various channel members. In many industries, these developments resulted in the traditional wholesale pricing arrangement being replaced by Revenue-Sharing Contracts (RSC). The paper investigates a manufacturer-retailer framework where the manufacturer sells a base product in two different formats: a tangible product sold directly to consumers and a digital format sold via the retailer. The latter also sells an optional contingent product (device). We find that the presence of the contingent product leads to a higher retail price for the digital base product and negatively affects the demand for the tangible product format. Furthermore we obtain that under wholesale pricing the profitability of the digital product is hurt by a double marginalization effect. The resulting too low digital product quantity also has a detrimental effect on the contingent product profit. For this reason the retailer prefers RSC almost always. The manufacturer is interested by its implementation only if it receives a sufficiently large part of the digital product revenue.

Key Words: M-commerce; Marketing channels, Pricing, Contingent products, Revenue-sharing contract.

1 Introduction

The proliferation of transaction websites, the growth of product digitization and the expansion of m-commerce¹ are some of the major retail developments driven by the digital revolution in the last two decades. Progress in Internet connectivity and in Information Technologies (IT) revolutionized retail by adding new online marketplaces, where products and services are made available to consumers in new formats. The introduction of digital formats (i.e., e-products) such as music MP3s and e-books that can be instantly purchased and consumed on mobile devices illustrates this revolution in some industries (Grewal et al., 2010).² In 2012 the Boston Consulting Group issued a series of articles on the "disruption" caused by e-commerce and emphasized the necessity for suppliers and retailers to "embrace these changes" and to adapt their strategies as soon as possible.³ Meanwhile, the phenomena of m-commerce has created new business development opportunities for the launch of handheld devices that facilitate users' experience when consuming digital products (Kalakota and Robinson, 2000).⁴ Some of these accessories are multifunctional (e.g., smartphones, smartwatches, or tablets), while others take the form of dedicated-for-use devices that allow consumers access only to a specific digital content (e.g., e-Reader and iPod). The latter attract consumers because of their light weight, their ease of use, and their lower prices (compared to multifunctional devices).⁵ These devices are often made available for purchase on the same online channels providing the digital content. In particular, many online retailers have taken advantage of the interdependencies between digital-product and mobile-device demands by introducing their own mobile devices. Many examples of such developments have been observed in the marketplace: Apple, a leader in the distribution of MP3 music launched the iPod and made it available via its iTunes library. Amazon launched its own reading device (i.e., Kindle) to take advantage of its position as a leader in the e-book market.⁶

The objective of this study is to examine the impact of such developments on the pricing strategies and profits of firms involved in a distribution channel where products are available in both tangible and digital formats: the latter is sold via an online retailer that, at the same time, distributes a mobile device dedicated to the use of the digital product.

Two recent literature reviews on channels (Watson et al., 2015; Krafft et al., 2015) stressed the importance of introducing the impact of technological development into future studies analyzing interactions in marketing channels. According to Watson et al. (2015), the development of e-commerce technologies (including access to mobile devices) is the trend that will exert the most fundamental change on future channel structures and strategies. Krafft et al. (2015) conducted a series of interviews with experts in marketing channel research to confirm that this trend is one of the most important issues requiring study in future research on channels. They report that, up until now, only a few studies have covered this topic.

During the early stages of the digital revolution, Kalakota and Robinson (2000) announced that firms should pay attention to these changes, and more particularly to their implications on the emergence of new business models. The authors urged managers to replace their traditional ways of doing business by adopting an e-business perspective where decisions are adapted to the digital world. Indeed, (i) changes in product formats, (ii) omnichannel distribution, and (iii) the emergence of new ways to consume digital products entail

¹I.e., mobile commerce. This development allows consumers to access a huge offering of products and services on wireless devices, anywhere and anytime.

²Cultural and informational products (e.g., books, newspapers, music, films), software and services (e.g., travel and hospitality) are among the industries that most strongly experienced the e-commerce revolution. In many industries, the online marketplace replaced traditional channels (e.g., nowadays, airplane tickets are mainly sold in digital format either by airline companies or by online travel agencies such as Expedia).

³Bellaïche et al. (2012). "Digital's Disruption of Consumer Goods and Retail".

⁴eMarketer reports that retail e-commerce sales worldwide reached \$1.61 trillion in 2015, which corresponds to 7.4% of total retail market. In the US market, retail e-commerce sales are expected to represent more than 10% of total retail sales in 2019. According to the same report, almost 40% of ecommerce sales are expected to be done on a mobile device. Source: Liu, C. (December, 2015) "Worldwide Retail Ecommerce Sales: eMarketer's Updated Estimates and Forecast Through 2019."

⁵An eMarketer report on US digital users states that the use of single-function devices such as e-readers will continue to grow in the coming years despite the presence of multifunctional devices (e.g., tablets and smartphones) because they have a longer battery life, non-glare screens, and lower prices. Source: Liu C. (February, 2016). "US Digital Users: The eMarketer Forecast for 2016" (p. 7).

⁶Before Apple's entry into this industry in 2010, Amazon used to account for 80% of the US e-book market. Source: *Wall Street Journal* Online (last visited Jan. 14, 2015) April 1, 2010.

changes in the other marketing variables, and especially on product pricing strategies (Kannan et al., 2009; Grewal et al., 2010). These changes affect not only the digital format but also its tangible counterpart. Because variable production costs are negligible for digital products, and suppliers are often dependent on major online retailers for the distribution of their products, the new pricing strategies are no longer based on costs, while at the same time, the balance of power between suppliers and e-tailers over the control of retail price decisions is changing. The traditional wholesale pricing model, whereby retailers used to purchase products at a cost (i.e., the wholesale price) and then set their retail prices by adding their profit margins, is being replaced in the most strongly impacted industries by agency pricing contracts, in which the retail price control is given to suppliers, while retailers receive a commission on sales in the form of a Revenue-Sharing Contract (RSC).⁷

The shift of control over retail prices in channels is not always easily accepted by vertical channel members. Many industries have experienced huge conflicts because of these changes. Suppliers have accused retailers of selling the digital content at very low prices in order to accelerate the adoption of their mobile devices (i.e., loss-leader pricing), and suppliers forced their retailers to move to an RSC.⁸ However, retailers wanted to keep control over retail prices and, as justification for their low retail prices, mentioned consumer welfare and the low cost of digital products.

For example, Amazon, who used to control e-book retail prices, disapproved of a change in business model that Macmillan, its content supplier, wanted to implement as an alternative to the traditional wholesale arrangement. Amazon reacted by ceasing all sales of Macmillan books on its e-Library. However, it was forced to capitulate two days later, when it accepted the new pricing contract. One of the reasons for this capitulation was that, because Macmillan is a major publisher, its removal from the Amazon online library significantly reduced the number of e-books on offer. This also negatively impacted its e-reader sales.⁹

The Macmillan-Amazon dispute over the choice of a pricing model was not unique, and similar conflicts have been documented in, e.g., the entertainment industry (digital music and films). Indeed, music providers (Universal Music Group and Warner Music Group) took issue with Apple when the retailer introduced the iPod and decided to charge a flat 99 cent rate for all songs bought on iTunes.¹⁰ In another dispute, this time between Apple and NBC Universal, the latter clearly announced that Apple's practice of loss-leader pricing was hurting the other channel members' interests. We expect similar conflicts to emerge in other industries affected by the digital wave, such as the video game industry.

Broadly, the above stories take place in a context with the following features: (a) there are two firms (a manufacturer and an e-retailer) engaged in a vertical relationship; (b) the manufacturer is offering a frequently purchased product in two different formats (digital and tangible) through two marketing channels (e-retailer and direct distribution); (c) the two formats are partially substitutable, and hence the e-retailer is simultaneously a partner of the manufacturer in the e-channel and its competitor in the product market; (d) in addition, the e-retailer sells a durable product (a device). Demand for this durable product is positively affected by sales of the digital product, while it is useless to consumers who do not buy this digital product's format. It is important to note that consumers can also use other devices for the consumption of the digital format. The device has a unilateral form of demand interdependency with the digital product and is called "an optional contingent product."

In this study, we want to examine the issue of pricing in vertical channel structures when a base product is sold in the marketplace in two competing formats via a traditional and an online channel and when a contingent product is made available by the online retailer. Retaining a marketing channel having the

⁷Cultural and informative industries are among those most affected by the digital wave and that have observed similar changes in their pricing models (i.e., a move from a wholesale business model to a revenue-sharing contract).

⁸Many national brand manufacturers disapprove of retailers' strategy of using their brands as loss-leaders because consumers might infer a poor quality from these low prices, or because specialty stores facing high competition from discount stores could cease to distribute their products (Tellis, 1986). In markets where products are available in digital and tangible formats, the reduced prices of the digital format may contribute to the decrease in consumers' willingness to pay for the other product format.

⁹Source: "Amazon's E-Book Price Reversal: A Mixed Blessing." *Business Week* website (last visited Jan. 19, 2016). Published February 2, 2010. MacMillan, D.

¹⁰Source: "Universal in Dispute With Apple Over iTunes." *The New York Times* website (last visited Jan. 14, 2015). Published July 2, 2007. Leeds, Jeff.

features (a)–(d) described above, we wish to provide answers to the following empirically and theoretically relevant questions:

1. Assuming that the channel adopts a wholesale pricing model, then
 - (a) What are the equilibrium prices and outcomes?
 - (b) What is the impact of offering the two product formats on these strategies?
 - (c) How does the presence of a contingent product affect these pricing strategies?
 - (d) Under what circumstances is loss-leader pricing an optimal strategy for the retailer?
2. Assuming an RSC is envisioned, then
 - (a) Under what circumstances would the manufacturer be indeed interested in implementing it?
 - (b) Among those cases that suit the leader (manufacturer), are there instances that also suit the retailer and/or consumer?

To answer these questions, we develop a parsimonious model retaining the features (a)–(d) mentioned above, and we characterize and compare the equilibrium strategies in the two business models, namely, the wholesale pricing arrangement and an RSC.

We mention from the outset that our context of pricing products in a marketing channel intrinsically differs from the highly studied cases of (i) pricing in an oligopoly (where the focus is on horizontal strategic interactions between a few firms competing with different brands in the same product category); (ii) pricing of substitute products in marketing channels (where the focus is on vertical strategic interactions between manufacturers and retailers, with possible horizontal interactions at the manufacturing and/or retailing layer(s)); and (iii) pricing of complementary products marketed by the *same* firm (i.e., product-line pricing). In the last case, it is clearly beneficial for the firm to control all the prices in the product line since decisions will be automatically coordinated by the same decision maker. When products are controlled by separate firms involved in a vertical structure, managers will not necessarily adopt the same strategies, because they will each have individual objectives.

In our setting, the base product is the frequently purchased product being sold in digital format by the manufacturer through its retailer, which also offers the contingent durable product. Hence, there is no valid reason for the manufacturer to heavily discount the price of the digital product, especially because this will hurt sales of the other product's format. However, there is a reason for the retailer to do so (i.e., to use a loss-leader pricing strategy), namely, to boost sales of the high-margin contingent product. Both the manufacturer and the retailer act as a category manager that fixes each product's price by taking into account the demand interdependencies within the product mix it handles. To be able to use the digital product as a springboard to increase sales of the contingent one, the retailer needs the manufacturer to provide the base product at a sufficiently low wholesale price. This may be too much to ask the manufacturer, especially if it sells directly to consumers a substitute version of the product (i.e., the tangible format), which could be subject to a cannibalization effect by the digital format. This means that all the ingredients for a conflict in the channel are in place.

Our first main result relates to the application of a loss-leader pricing strategy by the retailer. The idea is that the retailer sells the e-product at a loss in order to raise demand of the contingent product. We derive an explicit condition under which applying a loss-leader pricing strategy is optimal. Remarkable in this respect is that the profitability of the e-product market has contradictory effects. On the one hand, there is the expected effect that, when this market is not so profitable, the retailer concentrates on making profits in the contingent-product market. To do so, it enlarges contingent-product demand by increasing the e-product quantity, which reduces the e-product price so much that it falls below the wholesale price, implying loss-leader pricing. However, on the other hand, there is the effect that a less profitable e-product results from a low demand for this product, meaning that there are fewer potential buyers for the contingent product. This in turn leads to a less profitable contingent-product market. In such a case, loss-leader pricing makes less sense.

The second major result relates to the effect of changing the business model from wholesale to RSC. This as such ends the possibility for the retailer to apply a loss-leader strategy when pricing the digital-product format, because, within the RSC, the manufacturer has control over this retail price. Interestingly, we find that, under the RSC, the manufacturer fixes a lower retail price for the digital-format product (compared to its counterpart in the wholesale pricing arrangement), whereas the price of the tangible-product format is almost unchanged. The reason is that, under wholesale pricing, the digital product price is too high from an efficiency point of view. This is due to a double marginalization effect: first the manufacturer fixes the wholesale price such that it has a sufficient profit margin. Then the retailer fixes the sales price for the digital product such that it has some amount of margin. Hence, double marginalization.

The too high digital-product price under wholesale pricing implies that the digital product quantity is too low. This not only hurts digital-product profits, but it also reduces demand for the contingent product (if fewer digital products are around, fewer devices are needed to consume them), and thus, it also hurts contingent-product profits. For this reason the retailer almost always prefers RSC above wholesale pricing.

RSC in this framework requires the choice of the e-product revenue share that accrues to the retailer. We obviously obtain that the manufacturer will only push for an RSC if it gets a sufficiently large part of the digital-product revenue. Interestingly, in those cases where the manufacturer implements an RSC, the retailer also achieves a better payoff than in the wholesale price model. In many cases, implementing the RSC business model is also beneficial for the consumers.

The rest of the paper is organized as follows. Section 2 reviews the related literature, whereas Section 3 develops the model. Section 4 provides the results in the case of a wholesale pricing arrangement. In Section 5, we compute prices and profits under an RSC and compare these results with those obtained under Section 4. Finally, Section 6 concludes the paper.

2 Literature review

Our work is related to two literature streams, namely, product-mix pricing and pricing in vertical marketing channels. We review the relevant papers in these two streams and point out our contribution.

2.1 Product-mix pricing

As the name suggests, product-mix pricing refers to strategies and tactics used to exploit mutual dependencies between brands, products, or items (Tellis, 1986). Price bundling, customer-value pricing (premium pricing), and complementary-product pricing (including captive pricing, two-part tariff in services, and loss leadership) are examples of strategies used for pricing consumer products.

Peterson and Mahajan (1978) propose a classification of products based on the interdependency of their demands. Two products are said to be

Independent when the sales of one product are not affected, in a transparent way, by the sales of the other.

Substitutes when an increase in the sales of one product leads to a decrease in the sales of the other. Competing brands in any product category are typically substitutes. Different formats of the same product can also be considered as substitutes since they belong to the same product line.¹¹

Complements when the sales of one product positively affect the sales of the other because there is an associative “natural, functional relationship” between them and this relationship is symmetric (or bilateral). A typical example is a hamburger and french fries.

Contingent: when the sales of one product are conditional on the sales of another product. Mahajan and Muller (1991) distinguish between *captive contingent products* and *optional contingent products*. In the former case, we have bilateral or symmetric contingency, that is, neither product can be used without the other (e.g., a printer and its ink cartridges). In the case of optional contingent products, the

¹¹Some studies consider that the various formats of the same product could also be perceived as complements, depending on the utility that consumers derive from the product form (Kannan et al., 2009).

contingency is unilateral or asymmetric (e.g., a cell phone and its accessories). Obviously, a consumer would only consider buying the contingent product (accessories) if she buys (or has bought) the base product (cell phone), and a consumer who has bought the base product could use it without necessarily possessing the contingent one.

In most popular examples of contingent products, e.g., printer/ink cartridges and razor/blades, the base product is the durable one, and the contingent product is the repeat-purchase product (Bayus, 1987). In the example of the digital industries, it is the other way around. Indeed, no consumer would purchase an e-reader or an MP3 music player if e-books or MP3 music were not provided in the market.

Noble and Gruca (1999) surveyed 270 managers in order to identify the determinants of their pricing strategies. They found that complementary-product pricing is mainly used when the profitability of the contingent product is high. In such a context, the base product (printer, razor) is sold at cost or even given away for free, and money is made from the captive contingent product (ink cartridges, blades). This strategy, which is referred to as loss-leader pricing or the razor-and-blades business model,¹² is now practiced in many industries, but typically, for products belonging to the same product line and sold by the same firm (Kotler, 1988). The videogame industry for example is a heavy user of such a pricing strategy. The consoles are sold at a loss, and profits are generated from software and accessories sold by the same company. Mahajan and Muller (1991) obtain that the prices of the base and contingent product are lower when the two products are managed by the same company, relative to the case where they are controlled by two separate firms. As in Mahajan and Muller (1991), our study investigates the pricing of base and optional contingent products, however with one huge difference, that is, in our case the two independent firms interact in a vertical distribution structure where double marginalization could play a key role. A second main difference with respect to Mahajan and Muller (1991) is that our model considers the competition between two product formats sold in two channels: a direct and an online one.

2.2 Pricing in vertical marketing channels

This literature addresses multiple questions related to the pricing of products and services when this decision is controlled by agents interacting in a vertical distribution structure. One stream of research investigates the issue of channel coordination. Its main objective is to identify the equilibrium prices of a particular channel setting, or the best channel configuration leading to channel profit maximization (i.e., vertical integration or decentralization).¹³ Some of these studies are devoted to the design of coordination mechanisms and incentives that allow independent channel members to implement the prices resulting from cooperation (i.e., vertical integration). Revenue sharing is one of these mechanisms. Since the RSC links the channel members' individual outcomes to the joint outcome of a centralized channel according to the sharing rule, it allows channel members to reach coordination while remaining independent firms (Cachon, 2003; Cachon and Larivière, 2005).

There is a significant literature in marketing and operations management dealing with channel coordination and RSC, and reviewing it is beyond the objective of this paper.¹⁴ To the best of our knowledge, only a few papers study the coordination of pricing decisions in vertical structures where products are complementary, namely, Coughlan (1987), Moorthy (1988), Wang (2006), Dong et al. (2009) and Cai et al. (2012). However, none of them address the particular case of contingency in products' demands, and, more importantly, their focuses are very different from ours. Indeed, Coughlan (1987) examines different channel arrangements to distribute complementary products, and assesses how coordination at different levels could increase profitability. Moorthy (1988) examines the impact of different types of strategic interactions between channel members on the efficiency of a channel involving two dyads. He proves that, when the degree of demand complementarity between the manufacturers' products is high, each manufacturer is better off under decentralization when the other manufacturer chooses decentralization. Wang (2006) considers a setting with multiple manufacturers producing perfectly complementary products, that is, products that must

¹²In reference to Gillette, a company that has often used this price strategy.

¹³Issues related to the choice between online and/or traditional channels (i.e., omnichannel distribution) belong to this research stream. Our research differs from this literature since we examine an established channel, and our objective is to compare two pricing models for this given channel configuration.

¹⁴For an overview of the main results in non-competitive channels, the interested reader is referred to the recent survey by Ingene et al. (2012) or to Cachon (2003).

be purchased and consumed jointly. Although one retailer may be involved in the distribution system, the main issue is the determination by each manufacturer of the output and retail price for its product before the start of the selling season and in the presence of demand uncertainty. Dong et al. (2009) consider a bilateral monopoly where the manufacturer sells its line of products through an exclusive retailer and where the channel members have to decide whether they should implement product-line pricing or not.¹⁵ Finally, Cai et al. (2012) study the impact of exclusive channels and revenue sharing in a competitive channel where complementary goods (a product and a related service) are present. The manufacturer of each product chooses between having an exclusive service with one retailer or selling through both retailers who compete for the service.

A second topic in this literature stream investigates the impact of using various pricing models on prices, sales, and on channel members profits. Abhishek et al. (2013) belongs to this stream. The authors study the case where a manufacturer sells its product either directly to consumers (via a traditional channel) or via an e-channel where two retailers can choose to adopt a wholesale pricing model or an RSC. The authors compute and compare profits and prices under three configurations: (i) when both retailers implement a wholesale arrangement, (ii) when an RSC is adopted by both retailers, and (iii) a hybrid case where one of the retailers implements an RSC and the other one a wholesale arrangement. One of their results states that retailers' preferences between the two channel arrangements depends on the cross-effect between demands in both channels. When demand in the e-channel has a negative (positive) effect on the traditional channel's demand, the retailers prefer the RSC (a wholesale pricing arrangement). A second result states that an RSC leads to lower retail prices and higher consumer welfare, but that, when the retailers benefit from a positive externality (e.g. by selling a product that is complementary to the existing one), the opposite result is observed, i.e., retailers will prefer the wholesale pricing arrangement and retail prices are higher under the RSC.

Dantas et al. (2014) investigate pricing strategies in a channel where a unique manufacturer sells two competing product formats (paper book and e-book) via a single retailer. They compare firms' pricing strategies and profits under the wholesale and the agency-pricing (i.e., RSC) models and identify conditions under which the agency model can be implemented. The authors find that both products' retail prices are lower under revenue-sharing, and this business model is Pareto-improving when the revenue-sharing parameter, which represents the part of the revenue that goes to the retailer, is selected in the interval $[1/4, 1/2]$.

Gaudin and White (2014) identify the conditions under which the move from wholesale pricing to an RSC in the e-book industry contributes to the increase of e-book retail prices. They use a general demand system where the base product is available in one format, and consider two polar cases concerning the contingent product: either that consuming e-books requires use of the contingent product, or that there is perfect competition on the market for the contingent product, leading to a price equal to zero (no marginal costs). Gaudin and White (2014) mention that studying an oligopolistic model is a natural, yet challenging, extension of their work (see p.21), and this is exactly what we do. In addition, where they focus on the vertical merits of wholesale and RSC models, we also consider the horizontal competition concerning the sales of both product formats by the manufacturer and retailer, respectively.

As mentioned above, our study belongs to this research stream. However, we differ from this literature mainly by analyzing a marketing channel where we consider the interactions between three products (i.e., the two substitutable product formats and the optional contingent product), while they are controlled by different agents in the vertical distribution channel.¹⁶

¹⁵A firm using Product-Line Pricing (PLP) maximizes its profit by considering the interdependencies between all the products in the product line. In the scenario where the firm does not practice PLP, the authors consider that each product's price is determined independently from the others by distinct product managers.

¹⁶Only Abhishek et al. (2013) examined the case where the product is available in two formats and take into account the impact of the mobile-devices market on the pricing strategies of firms in the channel. Abhishek et al. (2013) considers two retailers, where we have just one. On the other hand, where Abhishek et al. (2013) fixes the price of the physical product to one, and models the effect of sales of the e-product on sales of the physical product exogenously, our model has a consumer utility function containing all products. Consequently, we get a demand system with endogenous dependencies between the demands of all products.

Discussing the different relationships between products, Shocker et al. (2004, p. 28) stated that “Research is needed to understand more complex cases in which contingencies and interactions among categories are present.” Our paper not only responds to this call, but also contributes to the literature in several other ways:

1. It extends the literature on the pricing of contingent products by looking at the case where these products are controlled by two *different* entities.
2. It extends the current modeling literature in marketing channels to digital-industry settings. We believe our model will shed light on equilibrium pricing in such industries, which will be of increasing importance in the economy. Our results give guidelines to each stakeholder in online distribution channels when it comes to elaborating its pricing strategy.
3. It offers some insight into the problem of which business model, that is, wholesale pricing or RSC, is better from the point of view of the different stakeholders (firms and consumers) where members control three products, two of them being characterized by contingency in demand, a case that has not been considered in the existing literature.

3 Models

This section introduces the wholesale pricing and RSC models. We consider a distribution channel composed of one manufacturer, player M , and one e-retailer (often called retailer), player R . The manufacturer sells a frequently purchased product, the base product b , in two different ways and formats: directly to the consumers and via the retailer. We distinguish between two product formats by letting bd refer to the version sold directly to the consumer, and bi to the version sold indirectly, that is, through the retailer. For product bi to be used, the consumer has the option to employ a dedicated-for-this-use durable product c (the contingent product), which is provided by the same retailer R . The price of a product is denoted P_x and the demand (quantity) Q_x with $x = bd, bi, c$. The vectors of prices and demands are $Q = (Q_{bd}, Q_c, Q_{bi})$ and $P = (P_{bd}, P_c, P_{bi})$, respectively. The wholesale price of product bi is denoted w_{bi} .

We denote by $U(Q_{bd}, Q_c, Q_{bi})$ the utility that the representative consumer derives from the three products. We make the following assumptions:

- A1:** If $Q_{bi} = 0$, then $Q_c = 0$.
- A2:** The utility function $U(Q_{bd}, Q_c, Q_{bi})$ is concave and increasing in its three arguments.
- A3:** The e-retailer purchases product c from its supplier at zero cost.
- A4:** The maximum willingness to pay for product bd is higher than or equal to the maximum willingness to pay for product bi .
- A5:** The direct-price effect on demand is larger than the cross-price effect.

Assumption A1 formally states the idea that the (optional) contingent product is useless if there is no demand for the digital format of the base product. Assumptions A2 and A5 are standard in economics. Assumption A3 is equivalent to saying that the supplier of the contingent product does not play any strategic role in our setting. Currently, it is empirically true that a digital product is typically sold at a lower price than its physical counterpart. Assumption A4 is a mild allusion to this empirical observation, that is, we do not require the equilibrium price P_{bd} to be necessarily higher than P_{bi} , but only the maximum willingness to pay to be ranked as stated.

3.1 Demand system

To derive the inverse demand system, we set up the problem of the representative consumer who needs to solve

$$\max_{Q_{bd}, Q_c, Q_{bi}} (U(Q_{bd}, Q_c, Q_{bi}) - P_{bd}Q_{bd} - P_cQ_c - P_{bi}Q_{bi}). \quad (1)$$

To proceed further, we must choose a functional form for the consumer utility function. For tractability reasons, we keep the form of the utility function quadratic as much as possible. However, in order to make sure that no utility is obtained from Q_c whenever $Q_{bi} = 0$ (Assumption A1), one third-order term is needed:

$$U(Q_{bd}, Q_c, Q_{bi}) = \lambda_{bd}Q_{bd} + \lambda_c Q_c Q_{bi} + \lambda_{bi}Q_{bi} - \frac{1}{2} \left(\theta_{bd} (Q_{bd})^2 + \theta_c (Q_c)^2 Q_{bi} + \theta_{bi} (Q_{bi})^2 + 2\mu Q_{bd}Q_{bi} \right), \quad (2)$$

in which λ_{bd} , λ_c , λ_{bi} , θ_{bd} , θ_c , θ_{bi} , and μ are positive constants. The parameter μ measures the degree of substitution between the two base product formats, i.e., the term $-\mu Q_{bd}Q_{bi}$ measures the loss in utility of the manufacturer's (retailer's) base product format due to the presence of the retailer's (manufacturer's) base product format. The reason is that the consumer can alternatively use the retailer's (manufacturer's) base product format. The following proposition introduces some restrictions on the parameter values to satisfy assumption A2.

Proposition 1 *The utility function is concave, provided the following conditions are satisfied:*

$$\theta_{bd}\theta_{bi} - \mu^2 > 0, \quad (3)$$

$$(\lambda_c - \theta_c Q_c)^2 < \frac{\theta_{bd}\theta_{bi} - \mu^2}{\theta_{bd}} \theta_c Q_{bi}. \quad (4)$$

Proof. See Appendix. □

Solving problem (1), where the utility function is given by (2), results in the following first-order conditions, which are at the same time the inverse demand functions:

$$P_{bd} = \lambda_{bd} - \theta_{bd}Q_{bd} - \mu Q_{bi}, \quad (5)$$

$$P_c = (\lambda_c - \theta_c Q_c) Q_{bi}, \quad (6)$$

$$P_{bi} = \lambda_{bi} - \theta_{bi}Q_{bi} - \mu Q_{bd} + \lambda_c Q_c - \frac{1}{2} \theta_c (Q_c)^2. \quad (7)$$

Assumptions A4 and A5 can be translated as follows in terms of parameter values:

$$\lambda_{bd} \geq \lambda_{bi}, \quad \theta_{bd} > \mu, \quad \theta_{bi} > \mu. \quad (8)$$

Back to the conditions in the above proposition, it is now clear that the first one is economically reasonable, because it is fulfilled if the quantity of a base product has a greater impact on its own price than the quantity of the other base product has on this price, that is, $\theta_{bd} > \mu$ and $\theta_{bi} > \mu$. (Note that A5 implies (3)). The second condition can only be checked afterwards, since this condition depends on Q_{bi} and Q_c . Finally, note that $\frac{\partial P_{bd}}{\partial Q_c} = \frac{\partial P_c}{\partial Q_{bd}} = 0$, which means that the relationship between products bd and c is not direct, but it is taken into account indirectly in the model via the interactions existing between products bd and bi in equations (5) and (7), and the interactions between products bi and c captured by equation (6).

We assume that producing the base product in bd format has a marginal cost C , with $C < \lambda_{bd}$, whereas this cost is zero for the bi format. The additional cost C reflects the fact that producing a tangible format for a product generates higher unit production costs than creating a digital product.

3.2 Pricing models

As stated in the introduction, we wish to characterize and compare the equilibrium solutions of two business pricing models, namely, wholesale pricing arrangements and RSC.

Wholesale pricing. Following a long tradition in the marketing-channel and supply-chain literature (see, e.g., the books by Ingene and Parry (2004), Jørgensen and Zaccour (2004), and the survey by Ingene et al. (2012)), we suppose that the game is played à la Stackelberg, with the manufacturer acting as leader and the retailer as follower. Consequently, the manufacturer first decides the wholesale price of

product bi and the quantity of product bd , that is, w_{bi} and Q_{bd} . Next the retailer sets the quantities Q_{bi} and Q_c . The optimization problems of the manufacturer and the retailer are as follows:

$$\begin{aligned}\pi_M &= \max_{w_{bi}, Q_{bd}} ((P_{bd} - C) Q_{bd} + w_{bi} Q_{bi}), \\ \pi_R &= \max_{Q_{bi}, Q_c} ((P_{bi} - w_{bi}) Q_{bi} + P_c Q_c).\end{aligned}$$

Substituting for the prices, we obtain

$$\pi_M = \max_{w_{bi}, Q_{bd}} ((\lambda_{bd} - \theta_{bd} Q_{bd} - \mu Q_{bi} - C) Q_{bd} + w_{bi} Q_{bi}), \quad (9)$$

$$\pi_R = \max_{Q_{bi}, Q_c} \left(\lambda_{bi} - \theta_{bi} Q_{bi} - \mu Q_{bd} + 2\lambda_c Q_c - \frac{3}{2} \theta_c (Q_c)^2 - w_{bi} \right) Q_{bi}. \quad (10)$$

Revenue-Sharing Contract. One of the main differences between wholesale pricing and RSC stems from the fact that the control over Q_{bi} passes from the retailer to the manufacturer (Abhishek et al. (2013)). In this case of revenue sharing in product bi market, the game is also sequential. We consider the variant with the manufacturer as leader, who chooses the quantities of both base products' formats, that is, Q_{bd} and Q_{bi} . Next, the retailer selects the contingent product quantity Q_c . The part $\alpha \in (0, 1)$ of the revenue of the base product bi that accrues to the retailer is exogenously fixed. The optimization problems of the retailer and the manufacturer are as follows:

$$\begin{aligned}\pi_M &= \max_{Q_{bi}, Q_{bd}} ((P_{bd} - C) Q_{bd} + (1 - \alpha) P_{bi} Q_{bi}), \\ \pi_R &= \max_{Q_c} (\alpha P_{bi} Q_{bi} + P_c Q_c).\end{aligned}$$

Substituting for the prices, we obtain the following expressions:

$$\pi_M = \max_{Q_{bi}, Q_{bd}} ((\lambda_{bd} - \theta_{bd} Q_{bd} - \mu Q_{bi} - C) Q_{bd} \quad (11)$$

$$+ (1 - \alpha) \left(\lambda_{bi} - \theta_{bi} Q_{bi} - \mu Q_{bd} + \lambda_c Q_c - \frac{1}{2} \theta_c (Q_c)^2 \right) Q_{bi}),$$

$$\pi_R = \max_{Q_c} \left(\alpha \lambda_{bi} - \alpha \theta_{bi} Q_{bi} - \alpha \mu Q_{bd} + (1 + \alpha) \lambda_c Q_c - \left(\frac{\alpha}{2} + 1 \right) \theta_c (Q_c)^2 \right) Q_{bi}. \quad (12)$$

4 Wholesale pricing model

Since the game is played à la Stackelberg, with the manufacturer acting as leader, we start by considering the retailer's problem. From (10) we obtain that product bi 's quantity Q_{bi} satisfies

$$\lambda_{bi} - 2\theta_{bi} Q_{bi} - \mu Q_{bd} + 2\lambda_c Q_c - \frac{3}{2} \theta_c (Q_c)^2 - w_{bi} = 0,$$

and that the contingent product's quantity Q_c equals

$$Q_c = \frac{2}{3} \frac{\lambda_c}{\theta_c}. \quad (13)$$

We see that there is strategic independence between Q_c and the manufacturer's decisions (Q_{bd} and w_{bi}), and strategic substitutability between Q_{bi} and Q_{bd} and w_{bi} . The fact that Q_c is independent of the manufacturer's decision variables is intuitive, as Q_c is provided by another supplier and does not compete directly with the manufacturer's products. The strategic substitutability between Q_{bi} and Q_{bd} is a by-product of the assumption that the two products are partially substitutable. The result that Q_{bi} is decreasing in w_{bi} is equivalent to stating that P_{bi} is increasing in w_{bi} . The fact that the retail price increases with the wholesale price is reminiscent of the double marginalization feature of vertical interaction and of the strategic substitutability between channel members' pricing decisions (Moorthy, 1988).

The following proposition characterizes the Stackelberg equilibrium strategy in the wholesale pricing model.

Proposition 2 Assuming that the solution is interior and the utility function concave,¹⁷ the unique interior Stackelberg pricing equilibrium strategy is given by

$$P_c = \lambda_c \left(\frac{\theta_{bd} (2\lambda_c^2 + 3\theta_c \lambda_{bi}) - 3\mu\theta_c (\lambda_{bd} - C)}{18\theta_c (2\theta_{bi}\theta_{bd} - \mu^2)} \right), \quad (14)$$

$$P_{bd} = \frac{1}{2} (\lambda_{bd} + C), \quad (15)$$

$$P_{bi} = \frac{2\lambda_c^2 (5\theta_{bd}\theta_{bi} - \mu^2) + 9\theta_c ((3\theta_{bd}\theta_{bi} - \mu^2) \lambda_{bi} - 9\mu\theta_c\theta_{bi} (\lambda_{bd} - C))}{18\theta_c (2\theta_{bi}\theta_{bd} - \mu^2)}, \quad (16)$$

$$w_{bi} = \frac{1}{2} \lambda_{bi} + \frac{1}{3} \frac{\lambda_c^2}{\theta_c}. \quad (17)$$

The players' profits are given by

$$\pi_M = \frac{\theta_{bd} (2\lambda_c^2 + 3\theta_c \lambda_{bi})^2 + 6\theta_c (\lambda_{bd} - C) (3\theta_c\theta_{bi} (\lambda_{bd} - C) - \mu (3\theta_c \lambda_{bi} + 2\lambda_c^2))}{36\theta_c^2 (2\theta_{bd}\theta_{bi} - \mu^2)}, \quad (18)$$

$$\pi_R = \theta_{bi} \frac{(2\lambda_c^2\theta_{bd} + 3\theta_c\theta_{bd}\lambda_{bi} + 3C\mu\theta_c - 3\mu\theta_c\lambda_{bd})^2}{36\theta_c^2 (\mu^2 - 2\theta_{bd}\theta_{bi})^2}. \quad (19)$$

Proof. See the Appendix. □

From the above proposition, we first conclude that the price of product bd is exactly half of the maximum willingness to pay of the consumer plus the production cost, and thus independent of μ , the parameter capturing the substitutability between the two formats. This can be explained by noting that increasing the parameter μ has contradictory effects on the bd product's price that exactly offset each other in equilibrium. First, from the inverse demand function (5), we obtain that the quantity of product bi has a larger negative effect on P_{bd} , resulting in a decrease of this price. Second, an increase of μ also has an effect on the quantities Q_{bd} and Q_{bi} (see expressions (36) and (37) in the Appendix), and the total negative effect results in a positive effect on the price P_{bd} such that it exactly counterbalances the first effect.

Second, we see that the wholesale price increases with λ_{bi} and λ_c , while it decreases with θ_c . We explain this by noting that the manufacturer will ask a higher wholesale price in case of a more profitable market for the base product sold by the retailer. This is the case when λ_{bi} is large, because the price is linearly increasing with this parameter. Also the bi product's demand is stimulated by sales of the contingent product, which, since $Q_c = \frac{2\lambda_c}{3\theta_c}$ (cf. (13)), is large when λ_c is large while θ_c is low.

To learn about the effects of the specific market characteristics that we consider here, we determine how the retailer's prices change when a substitute to product bi is not available or when there is no contingent product. The following proposition considers the scenario where product bd is absent.

¹⁷In the Appendix, we show that the conditions for an interior solution and for concavity of the utility function are given by

$$\begin{aligned} \frac{3\theta_c (\mu (\lambda_{bd} - C) - \theta_{bd}\lambda_{bi})}{2\theta_{bd}} &< \lambda_c^2 < \frac{3\theta_c (2\theta_{bi} (\lambda_{bd} - C) - \mu\lambda_{bi})}{2\mu}, \\ \lambda_c^2 &< \frac{9 (\theta_c (\theta_{bd}\lambda_{bi} - \mu (\lambda_{bd} - C))) (\theta_{bd}\theta_{bi} - \mu^2)}{2\theta_{bd} (2\theta_{bi}\theta_{bd} - \mu^2)}. \end{aligned}$$

For $\mu = 0$, the above conditions become

$$\begin{aligned} -\frac{3\theta_c\lambda_{bi}}{2} &< \lambda_c^2 < +\infty, \\ \lambda_c^2 &< \frac{9\theta_c\lambda_{bi}}{4}. \end{aligned}$$

This indicates that it is reasonable to think that the set of feasible parameter values is non-empty for μ sufficiently small.

Proposition 3 *The absence of an alternative product to bi leads to higher prices of products bi and c .*

Proof. Compute the differences

$$\begin{aligned} P_{bi} - P_{bi}(\mu = 0) &= \mu^2 \frac{3\mu\theta_c(\lambda_{bd} - C) - 3\theta_{bd}\theta_c\lambda_{bi} - 2\theta_{bd}\lambda_c^2}{12\theta_{bd}\theta_c(\mu^2 - 2\theta_{bd}\theta_{bi})} < 0, \\ P_c - P_c(\mu = 0) &= \mu\lambda_c \frac{6\theta_c\theta_{bd}(\lambda_{bd} - C) - 3\mu\theta_c\lambda_{bi} - 2\mu\lambda_c^2}{36\theta_{bi}\theta_c(\mu^2 - 2\theta_{bd}\theta_{bi})} < 0. \end{aligned}$$

Negativity in both cases follows from the positivity of the numerator, which is due to the assumption of an interior solution (see (44) in the Appendix), and the negativity of the denominator, which follows from (3). $\square$

The absence of competition on the base-product market implies that there is only one format available for consumption. This increases demand for the base product sold by the retailer, resulting in an increase of the corresponding price P_{bi} , compared to the case where an alternative exists. Since the contingent product is a strategic complement of product bi , demand for the contingent product also goes up, which implies that the contingent product's price P_c increases as well.

The following proposition establishes the effect of the availability of the contingent product.

Proposition 4 *The availability of product c leads to higher wholesale and retail prices of product bi , and higher demand. It leaves the price of product bd unchanged, but leads to a lower demand of this product.*

Proof. From the proof of Proposition 2, it is easy to see that in the absence of product c , we would have the following results:

$$\begin{aligned} \hat{P}_{bi} &= \frac{(3\theta_{bd}\theta_{bi} - \mu^2)\lambda_{bi} - \mu\theta_{bi}(\lambda_{bd} - C)}{2(2\theta_{bi}\theta_{bd} - \mu^2)} \\ \hat{w}_{bi} &= \frac{1}{2}\lambda_{bi}, \\ \hat{Q}_{bi} &= \frac{\theta_{bd}\lambda_{bi} - \mu(\lambda_{bd} - C)}{2(2\theta_{bi}\theta_{bd} - \mu^2)} \\ \hat{P}_{bd} &= \frac{1}{2}(C + \lambda_{bd}), \\ \hat{Q}_{bd} &= \frac{1}{(2\theta_{bi}\theta_{bd} - \mu^2)} \left(\theta_{bi}(\lambda_{bd} - C) - \frac{\mu}{2}\lambda_{bi} \right). \end{aligned}$$

It is straightforward to verify that

$$\begin{aligned} P_{bi} - \hat{P}_{bi} &= \frac{\lambda_c^2(5\theta_{bi}\theta_{bd} - \mu^2)}{9\theta_c(2\theta_{bi}\theta_{bd} - \mu^2)} > 0, \\ w_{bi} - \hat{w}_{bi} &= \frac{\lambda_c^2}{3\theta_c} > 0, \\ Q_{bi} - \hat{Q}_{bi} &= \frac{\theta_{bd}\lambda_c^2}{3\theta_c(2\theta_{bi}\theta_{bd} - \mu^2)} > 0, \\ P_{bd} &= \hat{P}_{bd}, \\ Q_{bd} - \hat{Q}_{bd} &= -\frac{\mu\lambda_c^2}{3\theta_c(2\theta_{bi}\theta_{bd} - \mu^2)} < 0. \end{aligned}$$

$\square$

The market for the base product sold by the retailer and the market for the contingent product each increase the other's size, due to the contingent interdependencies in both products' demands. Hence, it

makes sense that the presence of the contingent product results in an increase of the retail and wholesale prices, and the quantity of product bi . Since product bi and product bd are strategic substitutes, product bd suffers from the increased size of the market of product bi , which results in decreasing product bd 's quantity while leaving its price unchanged.

4.1 Loss-leader pricing

Under the wholesale pricing arrangement, the retailer controls not only the contingent product's retail price, but also the retail price of product bi . Then the retailer has the option to employ a loss-leader strategy where it sells the digital base product format at a loss to boost the sales of the optional contingent product. The next proposition presents the scenario under which the retailer finds it optimal to practice a loss-leader pricing strategy for product bi , that is, $P_{bi} < w_{bi}$.

Proposition 5 *If*

$$\lambda_{bi} < \frac{2\lambda_c^2}{9\theta_{bi}\theta_{bd}\theta_c} (\theta_{bd}\theta_{bi} - 2\mu^2) + \frac{\mu}{\theta_{bd}} (\lambda_{bd} - C), \quad (20)$$

then it is optimal for the retailer to adopt a loss-leader pricing strategy.

Proof. It suffices to compute $P_{bi} - w_{bi}$ from Proposition 2 to get the result. $\square$

Loss-leader pricing can be expected to occur when the profitability of the contingent product market is large. This is indeed reflected by expression (20): from this condition we first conclude that, given a sufficiently low value of μ , a loss-leader pricing strategy is optimal when the contingent product market is sufficiently profitable, thus, when λ_c is large and θ_c is low.

The profitability of the base product bi supplied by the retailer has two contradictory effects. First a low profitability for this market implies that the retailer concentrates on making profits from the contingent product market and thus pursues a loss-leader pricing strategy. Second a less profitable bi market results from low demand. This implies that the quantity of the bi product, Q_{bi} , is low, which means fewer potential buyers for the contingent product, and thus, in turn, a lower profitability for the contingent-product market. Thus a loss-leader strategy is less likely to be optimal. This explains the less straightforward effect of the base products' substitution parameter μ and of the price sensitivity parameter of base product bi , θ_{bi} , on the satisfaction of condition (20).

These two contradictory effects also play a role when we analyze the effect of the base product bd directly supplied on the market by the manufacturer. An increase of the price sensitivity parameter θ_{bd} reduces the profitability of the bd market, which makes this market less competitive than the bi market, thereby enhancing its profitability. This gives the two contradictory effects described in the previous paragraph. This explains why, when studying condition (20), we cannot conclude whether an increase of θ_{bd} supports the optimality of a loss-leader strategy. On the other hand, clearly, a loss-leader strategy is more likely to occur if the bd market is more profitable, in the sense of having a large maximum-willingness-to-pay parameter λ_{bd} and low unit costs C . This increases the quantity of base product bd . The resulting increased competition for the bi market reduces the bi product's price P_{bi} , making loss leader more likely, and its quantity Q_{bi} , making loss leader less likely since it reduces the number of potential consumers on the contingent product market. Apparently, the price effect is the largest here.

One clear conclusion that we can draw from the loss-leader condition is that loss-leader pricing is more likely to occur if λ_{bi} is low. This is because the price of the bi product positively depends on this parameter.

5 Revenue-sharing contract and comparison

Recall that, in this model, the retailer's share in the revenue of product bi is given by a parameter $\alpha \in (0, 1)$. One of our objectives is to find the range of values for α under which an RSC leads to higher profits for the manufacturer, who acts as leader in this channel, compared to the wholesale business model, which is taken

as the benchmark. Once this range of values is determined, we next check if it is also in the retailer's best interest to sign an RSC. Finally, we verify under what conditions the consumers would also prefer an RSC to a wholesale pricing arrangement.

In this RSC, the optimization problems are defined as follows:

$$\begin{aligned} \pi_M &= \max_{Q_{bi}, Q_{bd}} ((\lambda_{bd} - \theta_{bd} Q_{bd} - \mu Q_{bi} - C) Q_{bd} \\ &\quad + (1 - \alpha) \left(\lambda_{bi} - \theta_{bi} Q_{bi} - \mu Q_{bd} + \lambda_c Q_c - \frac{1}{2} \theta_c (Q_c)^2 \right) Q_{bi}), \end{aligned} \quad (21)$$

$$\pi_R = \max_{Q_c} \left(\alpha \lambda_{bi} - \alpha \theta_{bi} Q_{bi} - \alpha \mu Q_{bd} + (1 + \alpha) \lambda_c Q_c - \left(\frac{\alpha}{2} + 1 \right) \theta_c (Q_c)^2 \right). \quad (22)$$

As in the wholesale pricing arrangement, the manufacturer acts as leader in the RSC and chooses the quantities Q_{bd} and Q_{bi} of the b product. The retailer next chooses the quantity of Q_c . As usual in a leader-follower game, we begin by solving the follower problem. Considering retailer R 's optimization problem in (22), and assuming an interior solution, we obtain the following reaction function:

$$Q_c(Q_{bi}, Q_{bd}) = \frac{(1 + \alpha) \lambda_c}{(2 + \alpha) \theta_c}. \quad (23)$$

We observe that the contingent-product quantity is independent from the leader's decision variables, but does depend on the sharing parameter α , with

$$\frac{\partial Q_c}{\partial \alpha} = \frac{\lambda_c}{(2 + \alpha)^2 \theta_c} > 0,$$

that is, the higher the retailer's share of the revenue of product bi , the higher the quantity of the contingent product. Indeed, with a higher α , the retailer receives more of the revenue resulting from product bi sales. In such a case the retailer is more inclined to increase the willingness to pay for this product, which can be done by raising the quantity of the contingent product (see (7), from which we obtain that P_{bi} goes up with Q_c).

Substituting for Q_c from (23) in the manufacturer's optimization problem, we get

$$\begin{aligned} \pi_M &= \max_{Q_{bi}, Q_{bd}} ((\lambda_{bd} - \theta_{bd} Q_{bd} - \mu Q_{bi} - C) Q_{bd} \\ &\quad + (1 - \alpha) \left(\lambda_{bi} - \theta_{bi} Q_{bi} - \mu Q_{bd} + \frac{(1 + \alpha)(3 + \alpha) \lambda_c^2}{2(\alpha + 2)^2 \theta_c} \right) Q_{bi}). \end{aligned} \quad (24)$$

Assuming an interior solution, from the first-order optimality conditions we obtain the following quantities:¹⁸

$$\begin{aligned} Q_{bd} &= \frac{(1 - \alpha)}{4\theta_{bd}\theta_{bi}(1 - \alpha) - \mu^2(2 - \alpha)^2} \left(2\theta_{bi}(\lambda_{bd} - C) - \mu(2 - \alpha) \left(\lambda_{bi} + \frac{\lambda_c^2}{2\theta_c} \frac{(\alpha + 1)(\alpha + 3)}{(\alpha + 2)^2} \right) \right), \\ Q_{bi} &= \frac{2\theta_{bd}}{4\theta_{bd}\theta_{bi}(1 - \alpha) - \mu^2(2 - \alpha)^2} \left((1 - \alpha) \left(\lambda_{bi} + \frac{\lambda_c^2}{2\theta_c} \frac{(\alpha + 1)(\alpha + 3)}{(\alpha + 2)^2} \right) - \frac{\mu(2 - \alpha)(\lambda_{bd} - C)}{2\theta_{bd}} \right). \end{aligned}$$

Inserting the equilibrium quantities in the inverse demand and profit functions, we get very long expressions, which are by no means amenable to a qualitative analysis. Consequently, we resort to a numerical analysis

¹⁸If we assume that

$$4\theta_{bd}\theta_{bi}(1 - \alpha) - \mu^2(2 - \alpha)^2 > 0, \quad (25)$$

then the conditions for an interior solution are

$$\frac{\mu(2 - \alpha)}{2\theta_{bi}} \left(\lambda_{bi} + \frac{\lambda_c^2}{2\theta_c} \frac{(\alpha + 1)(\alpha + 3)}{(\alpha + 2)^2} \right) < \lambda_{bd} - C < \frac{2\theta_{bd}(1 - \alpha)}{\mu(2 - \alpha)} \left(\lambda_{bi} + \frac{\lambda_c^2}{2\theta_c} \frac{(\alpha + 1)(\alpha + 3)}{(\alpha + 2)^2} \right). \quad (26)$$

The condition in (25) is slightly more restrictive than the condition for concavity of the utility function, which is

$$4\theta_{bd}\theta_{bi} - \mu^2 > 0.$$

Further, condition (26) is always satisfied for $\mu = 0$ and thus for μ sufficiently small.

to answer our research questions 2a and 2b posed in the Introduction. The first task is then to define a benchmark case. The following parameter value constellation is such that we have solutions for both business models satisfying all assumptions and nonnegativity of prices, quantities, and profits:

$$\lambda_{bd} = 20, \lambda_{bi} = 9, \lambda_c = 2, \theta_{bd} = \theta_c = \theta_{bi} = 1, \mu = 0.5 \text{ and } C = 15.$$

Table 1 reports the results for the benchmark case under the two pricing models for different values of the revenue-sharing parameter α .¹⁹

Table 1: Results in the benchmark case

WP Model		RSC with different values of α								
α		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Q_c	1.33	NA	1.09	1.13	1.17	1.20	1.23	1.26	1.29	NA
Q_{bd}	1.19	NA	0.16	0.33	0.50	0.69	0.90	1.17	1.62	NA
Q_{bi}	2.62	NA	5.20	5.11	4.99	4.82	4.56	4.10	2.95	NA
w_{bi}	5.83	-	-	-	-	-	-	-	-	-
P_{bd}	17.50	NA	17.24	17.12	17.00	16.90	16.82	16.78	16.91	NA
P_c	1.75	NA	4.73	4.45	4.16	3.86	3.51	3.03	2.11	NA
P_{bi}	7.56	NA	5.30	5.35	5.41	5.51	5.69	6.04	6.99	NA
π_R	6.86	NA	10.68	13.22	15.65	17.92	19.89	21.16	19.20	NA
π_M	18.25	NA	22.43	19.82	17.21	14.60	12.02	9.51	7.21	NA
$\pi_{Channel}$	25.11	NA	33.12	33.05	32.86	32.52	31.92	30.67	26.41	NA
U	46.34	NA	44.29	46.89	49.40	51.80	54.02	55.84	55.96	NA

NA (Non Admissible). This symbol is used to denote cases where the level of α , combined with the values of the other model's parameters in the benchmark lead to results that violate at least one of the conditions related to the concavity of the utility function and the positivity of quantities and prices.

In the discussion below, we use the superscript *WP* to refer to the wholesale pricing model and superscript *RSC* for the revenue-sharing contract. We first of all observe that the *bi* product price is higher under the wholesale pricing model. This comes from the double marginalization effect, resulting from the vertical distribution structure of the *bi* product. To illustrate, let us, for the numerical parameter values of the benchmark case, calculate profits in absence of the vertical structure. Thus, consider just a framework where the manufacturer sells the product *bd* and the retailer sells product *bi*, which is not supplied to the retailer by the manufacturer in this case. Using (13) we obtain that the contingent-product quantity is given by

$$Q_c = 1.33.$$

Hence, we have the following Cournot problem, where player *M* maximizes

$$\max_{Q_{bd}} \left(\left(5 - Q_{bd} - \frac{1}{2} Q_{bi} \right) Q_{bd} \right),$$

whereas for player *R* we have

$$\max_{Q_{bi}} \left(\frac{35}{3} - Q_{bi} - \frac{1}{2} Q_{bd} \right) Q_{bi}.$$

This straightforwardly gives (superscript *C* stands for "Cournot")

$$Q_{bd}^C = 1.11, Q_{bi}^C = 5.56, P_{bd}^C = 16.11, P_{bi}^C = 5.56.$$

We conclude that, indeed, due to the double marginalization effect, the price in the *bi* market under wholesale pricing is too high, i.e., $P_{bi}^{WP} = 7.56 > P_{bi}^C = 5.56$, which results in a too-low quantity $Q_{bi}^{WP} = 2.62 < Q_{bi}^C = 5.56$. In fact, our numerical results also show that it holds that $Q_{bi}^{WP} < Q_{bi}^{RSC}$. This inefficiency has two negative effects. First, profit from the *bi* market is low, which hurts both the manufacturer and the retailer. Second, the low quantity of product *bi* has a negative effect on the demand for the contingent product.

¹⁹We do not investigate the extreme cases where $\alpha \in \{0, 1\}$ since these situations cannot be implemented. They imply that only one channel member takes up the total channel revenue.

Consequently, contingent-product profits are also low. This explains why profits are so low for the retailer under wholesale pricing, so that it would certainly prefer the RSC model, even when the retailer receives only 20% of the revenue of product bi .

Under wholesale pricing, the manufacturer is also hurt by the inefficiently low quantity Q_{bi}^{WP} , leading to too-low profits resulting from selling product bi to consumers via the retailer. On the other hand, the manufacturer also collects profits from selling product bd , and these profits are higher,²⁰ because the too-low quantity Q_{bi}^{WP} positively affects demand for product bd . Therefore, the manufacturer will always set a higher price under wholesale pricing, i.e., $P_{bd}^{WP} > P_{bd}^{RSC}$, as our numerical results confirm.

As a result, manufacturer profits under wholesale pricing are comparable to the profits under the RSC model. In particular, there exists an upper bound value on the sharing parameter, call it α_u , such that, for any $\alpha \geq \alpha_u$, wholesale profits are higher for the manufacturer compared to under RSC. In this benchmark case, we observe that α_u is in the interval $(0.3, 0.4)$. Consequently, the manufacturer will implement an RSC for α relatively low, that is, larger than 0.1 and lower than α_u . It follows that an RSC is profit Pareto improving in this case. Note that we do not have admissible solutions for $\alpha = 0.1$ and $\alpha = 0.9$, that is, when one player's share is too low.

Concerning the contingent-product market, we can prove analytically that it is less developed under the RSC, regardless of the revenue-sharing parameter value. Indeed,

$$Q_c^{WP} - Q_c^{RSC} = \frac{\lambda_c}{3\theta_c} \left(\frac{1 - \alpha}{2 + \alpha} \right) > 0.$$

This is because the retailer, who fixes the contingent product quantity, has a higher incentive to increase this quantity under wholesale pricing. The reason is that Q_c adds to the demand of product bi , and under wholesale pricing, the retailer receives all the profits from selling product bi , while they need to be shared under RSC. Still we have to keep in mind that, as stated above, contingent product profits are higher under RSC, because demand undergoes more stimulation there from the higher quantity of product bi . Hence, it has to hold that $P_c^{RSC} > P_c^{WP}$, which is confirmed by our numerical results.

Finally, we observe that consumer utility is increasing in α . This is first of all due to Q_{bd}^{RSC} being strictly increasing in α . For low values of α , the manufacturer receives a great deal of revenue from the bi market. Therefore, it keeps Q_{bd}^{RSC} low in order to stimulate demand of product bi . Furthermore, for higher values of α the retailer collects more of the bi -market profits. Therefore, for higher α , it is more inclined to stimulate demand for the bi product by raising the contingent product quantity Q_c . Both of these developments apparently more than compensate for the drop in Q_{bi} for rising α , which is because the manufacturer, who decides on Q_{bi} , receives a smaller portion of the corresponding revenue. In our benchmark case, a switch from wholesale pricing to a revenue-sharing contract with $\alpha = 0.3$ would benefit all market participants, i.e., consumers, manufacturer, and retailer.

The next step is to check the robustness of these results by varying the parameter values. To keep the number of simulations reasonable, we fix once and for all the values of some of the parameters, namely, $\theta_{bi} = \theta_{bd} = \theta_c = 1$, and $C = 15$ (as in the benchmark case). This is not a severe assumption, as we vary the other demand parameters, that is, μ , λ_{bi} , λ_{bd} and λ_c , whereas the cost parameter only has a quantitative impact.²¹ More specifically, we consider the following values for these parameters:

$$\begin{aligned} \mu &\in \{0.25, 0.5, 0.75\} \\ \lambda_{bd} &\in \{16, 17, 18, 19, 20, 21, 22, 23, 24, 25\}, \\ \lambda_{bi} &\in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}, \\ \lambda_c &\in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}, \\ \alpha &\in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}. \end{aligned}$$

²⁰With the exception of $\alpha = 0.8$, where the manufacturer's share of product bi revenue is so low that it mainly concentrates on the bd market.

²¹Actually, we examined the effects of these parameters on the results and found that they do not affect the results qualitatively.

This means that the Stackelberg equilibrium in the RSC was computed for 27,000 cases ($3 \times 10 \times 10 \times 10 \times 9 = 27,000$). For the wholesale model, the number is 3,000, as the sharing parameter α does not appear there. Note that the benchmark case belongs to the set of retained experiments.

To answer our research question 2a, namely, when is the manufacturer interested in implementing an RSC, Tables 2 and 3 report the percentages of cases where the manufacturer and consumers respectively are better off with an RSC than with wholesale pricing arrangements. In both tables, these percentages are computed after excluding the cases where the results violate one of the admissibility criteria, i.e., concavity of the utility function and nonnegativity of prices, demands and profits. These results can be summarized as follows:

1. The existence of an upper bound for α above which the manufacturer will not implement the RSC is confirmed, which of course makes sense because the share of the bi revenue the manufacturer receives decreases with α . In any event, the upper bound value is close to 0.3.²²
2. Even for a value of α as low as 0.1, it is far from granted that the manufacturer will find it optimal to push for RSC. This is because the retailer does not receive much of the bi revenue and therefore does not have much incentive to stimulate demand for product bi by raising the quantity of the contingent product. This also reduces the revenue of product bi for the manufacturer.
3. For all cases where an RSC leads to higher profits for the manufacturer, we make the following observations:
 - (a) For all parameter values, the channel's total profit under an RSC is also higher. Therefore, where the manufacturer receives higher profits under RSC, an RSC is more efficient than wholesale pricing. This is because, with very few exceptions, our simulations show that the retailer is always better off with an RSC than with wholesale pricing.²³ As explained above, the reason for this is that, especially under wholesale pricing, the retailer is hurt by the inefficiency in the bi market due to the double marginalization effect, which leads to insufficient quantity of product bi .
 - (b) All the statements regarding the prices of bi , bd and c in the benchmark case, remain valid for all parameter values. The conclusion is that an RSC leads to lower retail prices of products bi and bd and a higher price of the contingent product c .
 - (c) The consumer surplus highly depends on the value of μ , the parameter of substitutability between the two formats of the b product. For the highest retained value, that is, $\mu = 0.75$, the message is clear-cut: an RSC always hurts the consumer. In this case, the formats are highly substitutable, and, because under wholesale pricing, different firms decide about the retail prices of the different formats of product b , the resulting competition leads to relatively low prices benefitting the consumers. For $\mu \in \{0.25, 0.5\}$, the percentage is increasing in α and reaches its maximum at $\alpha = 0.4$. This arises because the manufacturer reduces Q_{bi} when α goes up, because it gets a lower share of the profits in the bi -product market.

To investigate the impact of λ_x , $x \in \{bd, bi, c\}$ on the percentage of cases where the RSC can be implemented, we set the value of α at 0.3, which corresponds in the benchmark to the case where the RSC is preferred by all parties, i.e., the retailer, the manufacturer, and the consumers. We found that the percentage of cases where an RSC is preferred by both the manufacturer and the retailer increases

- when λ_{bi} increases. This increases the willingness to pay for product bi . Then the double marginalization effect that makes the bi market more inefficient under wholesale pricing becomes bigger, so it is understandable that the RSC pricing model will be more preferable.
- when λ_c decreases. Under wholesale pricing, the quantity in the contingent product market is higher because the retailer is more inclined to stimulate demand on the bi market. This is because it can

²²Interestingly, this is in line with the revenue-sharing contract between Apple and MacMillan, which gives 30% to Apple.

²³For $\alpha = 0.1$, and when $\mu \in \{0.25, 0.5\}$, we found a percentage of simulations (around 10%) where retailer's profit is negatively impacted by the implementation of the RSC. These cases correspond to situations where the RSC is channel efficient but hurts the retailer. One can imagine the manufacturer offering the retailer a side payment to encourage its participation and avoid channel conflict.

Table 2: Percentages of cases where M 's profit is higher with an RSC

	$\mu = 0.25$	$\mu = 0.5$	$\mu = 0.75$
$\alpha = 0.1$	34.34	38.32	36.13
$\alpha = 0.2$	25.13	26.86	19.49
$\alpha = 0.3$	14.94	13.01	7.94
$\alpha = 0.4$	4.1	2.97	0
$\alpha = 0.5$	0	0	0
$\alpha = 0.6$	0	0	0
$\alpha = 0.7$	0	0	0
$\alpha = 0.8$	0	0	NA
$\alpha = 0.9$	0	0	NA

Table 3: Percentages of cases where consumer surplus is higher with an RSC

	$\mu = 0.25$	$\mu = 0.5$	$\mu = 0.75$
$\alpha = 0.1$	66.84	0	0
$\alpha = 0.2$	78.47	2.63	0
$\alpha = 0.3$	90.91	18.42	0
$\alpha = 0.4$	96.00	44.44	NA
$\alpha \in \{0.5, 0.6, 0.7, 0.8, 0.9\}$	NA	NA	NA

keep all profits on this market for itself, while, under an RSC, it has to share these profits with the manufacturer. When λ_c decreases the role of the contingent-product market is more minor, so that this effect in favor of wholesale pricing diminishes.

- does not have a clear-cut impact when λ_{bd} increases. If λ_{bd} goes up the willingness to pay for product bd increases. Under wholesale pricing, this market is more profitable because the quantity of the substitute product bi is lower due to the double marginalization effect. This implies that the manufacturer will increasingly appreciate wholesale pricing when λ_{bd} increases. However, this is the other way round for the retailer because a higher value of λ_{bd} increases the quantity of the product bd , which reduces profitability of the bi market. This explains why the effect of λ_{bd} on the relative appreciation of RSC by both market parties is less clear-cut.

6 Conclusions

In this study, we examined a vertical distribution structure where a monopolist manufacturer supplies a base product in two formats sold via two channels: a tangible product sold directly to consumers and a digital format sold via a single online retailer. The latter also offers an optional contingent product that consumers may employ to consume the digital version of the base product. Such a setting frequently occurs in digital industries. Obvious examples are the e-book and MP3 music markets (base products) with the e-reader and the iPod as optional contingent products.

In such a context channel members can choose between adopting a wholesale price arrangement, where the retailer controls the digital product's price, or implementing an RSC, which involves passing the control over the digital product's price from the retailer to the manufacturer.

Our objective is to provide answers on two main questions: The first one investigates the contingent product's impact on firms' pricing strategies when a wholesale pricing arrangement is adopted and explores the conditions under which the retailer, who controls the digital product's price, uses the digital product as a loss-leader. The second question investigates whether the different stakeholders in the channel (i.e., the manufacturer, the retailer, and consumers) are benefitted by the implementation of an RSC.

Our first result states that the presence of the contingent product leads to a higher retail price for the digital base product and negatively affects the demand for the tangible product format. Furthermore, using a loss-leader pricing strategy where the digital product is sold at a loss in order to boost sales of the contingent

product, does not always benefit the retailer. This result is explained by the fact that a less profitable digital-base-product market could also have harming effects on the contingent-product market.

On the implementation of an RSC, we identify an upper bound for the revenue-sharing parameter beyond which the manufacturer does not want an RSC because it earns too less on the digital product. The retailer, on the other hand, almost always has a preference for applying a RSC business model over the more traditional wholesale pricing arrangement. The reason is that, under wholesale pricing, the price of the digital product is too high due to a double marginalization effect: in selling to the retailer, the manufacturer likes to fix a positive profit margin, and the retailer wants the same when selling the digital product to consumers. This inefficiency not only leads to low profits for the digital products, but it also holds for the optional contingent product market. The latter effect can be explained as follows. The high digital product price implies that not too many of these products are sold to consumers. This in turn implies that demand for the optional contingent product will be low, because these products can only be used to consume the digital product.

This paper is an attempt to understand the relative value of business models like wholesale pricing and revenue-sharing contracts in the presence of base and optional contingent products, in a one-manufacturer, one-retailer supply chain. One meaningful extension is to consider competition in the contingent product market. Here, the conjecture is that competition would incite firms to reduce the contingent products' retail prices, which would contribute to an increase in demand for both the digital base and the contingent product. Then one could investigate if the channel efficiency can be improved by this competitive effect and how this affects the appreciation for the different business models.

In the digital-books industry for example, we observe that competition is now present in the e-reader market, following the arrival of Nook (by Barnes & Noble), Kobo (by Indigo), etc. Also, tablets are competitors to the e-readers that have multi-functional properties. Hence, their demand interdependencies with digital books are complementary rather than optional contingent, as it is the case for e-readers.²⁴ As usually expected in such a context, the retail price of e-readers went down. More than that, Amazon started to sell the Kindle as a loss-leader. The second reaction of firms controlling such devices was to add some features to e-readers that allow consumers to use them for different purposes. These developments bring up interesting research questions: (i) Does competition necessarily lead to the classical situation, where loss-leader pricing is reserved for the durable product (e.g., e-reader)? (ii) How long can an optional contingent product survive in the market, given that multi-functional products are around the corner?

Many experts in the industry consider that the fall of the "tangible" formats in the entertainment industry is mainly due to pirating. According to these experts, things could be different for the book industry if editors "provide a legitimate alternative to pirated materials" and if they adopt a well-thought-out pricing strategy. By doing so, they would allow e-books to play a complementary role rather than a substitute for print, since digital books could create a new market by attracting customers "who would not have purchased a traditional book but may be inclined to buy an e-book that costs less, offers additional features, and works on a digital device they already own."²⁵ A recent survey of US customers published by The Pew Research Center's Internet Project confirmed this analysis.²⁶

Taking stock of the above and the rapid technological developments in the digital book and other similar industries, we believe that the following research questions need additional attention: (i) Is there a future for tangible products when a digital alternative exists? (ii) What are the best pricing strategies for different formats of a product (book, music, etc.), where the digital product fulfills a complementary role instead of providing competition to the tangible product format, as analyzed in this paper? (iii) How does real or potential piracy affect pricing strategies?

²⁴Kindle sales moved from 11.6 to 12.5 million units between 2011 and 2012: an 8% increase. If we compare this to the 325% increase observed between 2009 and 2010 (the year when Nook, Kobo and iPad were launched), we can see the negative impact of competition on Amazon's dominant position in the e-reader market. Source: *eMarketer Report*, September 11, 2012.

²⁵Source: "Turning the Page: The Future of eBooks". A pwc report (2010, p3).

²⁶Source: "Ebook Readers Use Devices to Supplement, Not Replace, Printed Media." *eMarketer*. January 30, 2014.

7 Appendix

7.1 Proof of Proposition 1

$$U(Q_{bd}, Q_c, Q_{bi}) = \lambda_{bd}Q_{bd} + \lambda_c Q_c Q_{bi} + \lambda_{bi}Q_{bi} - \frac{1}{2} \left(\theta_{bd} (Q_{bd})^2 + \theta_c (Q_c)^2 Q_{bi} + \theta_{bi} (Q_{bi})^2 + 2\mu Q_{bd}Q_{bi} \right), \quad (27)$$

$$\frac{\partial U}{\partial Q_{bd}} = \lambda_{bd} - (\theta_{bd}Q_{bd} + \mu Q_{bi}) \quad (28)$$

$$\frac{\partial U}{\partial Q_c} = \lambda_c Q_{bi} - (\theta_c Q_c Q_{bi}), \quad (29)$$

$$\frac{\partial U}{\partial Q_{bi}} = \lambda_c Q_c + \lambda_{bi}Q_{bi} - \frac{1}{2} \left(\theta_c (Q_c)^2 + 2\theta_{bi}Q_{bi} + 2\mu Q_{bd} \right), \quad (30)$$

Concavity of the utility function requires that the determinant of the Hessian matrix be negative, i.e.,

$$\det \begin{pmatrix} -\theta_{bd} & 0 & -\mu \\ 0 & -\theta_c Q_{bi} & \lambda_c - \theta_c Q_c \\ -\mu & \lambda_c - \theta_c Q_c & -\theta_{bi} \end{pmatrix} = -(\theta_{bd}\theta_{bi} - \mu^2) \theta_c Q_{bi} + \theta_{bd} (\lambda_c - \theta_c Q_c)^2 < 0.$$

Concavity also requires that the diagonal elements of the Hessian matrix be negative, which is the case, and that

$$\begin{aligned} \det \begin{pmatrix} -\theta_{bd} & 0 \\ 0 & -\theta_c Q_{bi} \end{pmatrix} &= \theta_{bd}\theta_c Q_{bi} > 0, \\ \det \begin{pmatrix} -\theta_{bd} & -\mu \\ -\mu & -\theta_{bi} \end{pmatrix} &= \theta_{bd}\theta_{bi} - \mu^2 > 0, \\ \det \begin{pmatrix} -\theta_c Q_{bi} & \lambda_c - \theta_c Q_c \\ \lambda_c - \theta_c Q_c & -\theta_{bi} \end{pmatrix} &= \theta_{bi}\theta_c Q_{bi} - (\lambda_c - \theta_c Q_c)^2 > 0. \end{aligned}$$

So concavity of the utility function requires

$$\theta_{bd}\theta_{bi} - \mu^2 > 0,$$

and

$$(\lambda_c - \theta_c Q_c)^2 < \min \left(\frac{\theta_{bd}\theta_{bi} - \mu^2}{\theta_{bd}}, \theta_{bi} \right) \theta_c Q_{bi} = \frac{\theta_{bd}\theta_{bi} - \mu^2}{\theta_{bd}} \theta_c Q_{bi}.$$

7.2 Proof of Proposition 2

We start by solving the retailer's problem. The first-order conditions of player R are given by

$$\frac{\partial \pi_R}{\partial Q_{bi}} = \lambda_{bi} - 2\theta_{bi}Q_{bi} - \mu Q_{bd} + 2\lambda_c Q_c - \frac{3}{2} \theta_c (Q_c)^2 - w_{bi} = 0, \quad (31)$$

$$\frac{\partial \pi_R}{\partial Q_c} = (2\lambda_c - 3\theta_c Q_c) Q_{bi} = 0. \quad (32)$$

From (32) we get

$$Q_c = \frac{2}{3} \frac{\lambda_c}{\theta_c}. \quad (33)$$

Substitution of (33) into (31) gives

$$Q_{bi} = \frac{1}{2\theta_{bi}} \left(\lambda_{bi} - \mu Q_{bd} + \frac{2}{3} \frac{\lambda_c^2}{\theta_c} - w_{bi} \right). \quad (34)$$

We now move to the problem of the manufacturer who solves

$$\begin{aligned}\pi_M &= \max_{w_{bi}, Q_{bd}} ((\lambda_{bd} - \theta_{bd} Q_{bd} - \mu Q_{bi}(Q_{bd}, w_{bi})) Q_{bd} + w_{bi} Q_{bi}(Q_{bd}, w_{bi})) \\ &= \max_{w_{bi}, Q_{bd}} \left((\lambda_{bd} - \theta_{bd} Q_{bd}) Q_{bd} + (w_{bi} - \mu Q_{bd}) \frac{1}{2\theta_{bi}} \left(\lambda_{bi} - \mu Q_{bd} + \frac{2}{3} \frac{\lambda_c^2}{\theta_c} - w_{bi} \right) \right).\end{aligned}$$

The equilibrium wholesale price, w_{bi} , equals

$$w_{bi} = \frac{1}{2} \lambda_{bi} + \frac{1}{3} \frac{\lambda_c^2}{\theta_c}, \quad (35)$$

whereas the quantity for the base product bd is

$$Q_{bd} = \frac{1}{(2\theta_{bi}\theta_{bd} - \mu^2)} \left(\theta_{bi} (\lambda_{bd} - C) - \frac{\mu}{2} \lambda_{bi} - \frac{\mu \lambda_c^2}{3\theta_c} \right). \quad (36)$$

Next, we determine Q_{bi} by substitution of (35) and (36) into (34):

$$Q_{bi} = \frac{1}{6} \frac{\theta_{bd} (2\lambda_c^2 + 3\theta_c \lambda_{bi}) - 3\mu\theta_c (\lambda_{bd} - C)}{\theta_c (2\theta_{bi}\theta_{bd} - \mu^2)}. \quad (37)$$

It suffices to insert the values of Q_c , Q_{bd} and Q_{bi} in (5)-(7) to get the following prices:

$$P_{bd} = \frac{1}{2} (\lambda_{bd} + C), \quad (38)$$

$$P_c = \lambda_c \left(\frac{\theta_{bd} (2\lambda_c^2 + 3\theta_c \lambda_{bi}) - 3\mu\theta_c (\lambda_{bd} - C)}{18\theta_c (2\theta_{bi}\theta_{bd} - \mu^2)} \right), \quad (39)$$

$$P_{bi} = \frac{2\lambda_c^2 (5\theta_{bd}\theta_{bi} - \mu^2) + 9 (3\theta_{bd}\theta_{bi} - \mu^2) \theta_c \lambda_{bi} - 9\mu\theta_c \theta_{bi} (\lambda_{bd} - C)}{18\theta_c (2\theta_{bi}\theta_{bd} - \mu^2)} \quad (40)$$

Concavity requires the satisfaction of conditions (3)-(4). Substitute for (33) and (37) in (4) to obtain the condition in terms of only the parameters, that is,

$$\lambda_c^2 < \frac{9 (\theta_c (\theta_{bd} \lambda_{bi} - \mu (\lambda_{bd} - C))) (\theta_{bd} \theta_{bi} - \mu^2)}{2\theta_{bd} (2\theta_{bi}\theta_{bd} - \mu^2)}. \quad (41)$$

To have an interior solution, the following conditions must be satisfied:

$$Q_{bd} > 0 \Leftrightarrow 6\theta_{bi}\theta_c (\lambda_{bd} - C) - 3\mu\theta_c \lambda_{bi} - 2\mu\lambda_c^2 > 0, \quad (42)$$

$$Q_{bi} > 0 \Leftrightarrow \theta_{bd} (2\lambda_c^2 + 3\theta_c \lambda_{bi}) - 3\mu\theta_c (\lambda_{bd} - C) > 0. \quad (43)$$

The above two conditions are equivalent to

$$\frac{3\theta_c (\mu (\lambda_{bd} - C) - \theta_{bd} \lambda_{bi})}{2\theta_{bd}} < \lambda_c^2 < \frac{3\theta_c (2\theta_{bi} (\lambda_{bd} - C) - \mu \lambda_{bi})}{2\mu}. \quad (44)$$

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