CURRICULUM

Mathematics 314

secondary school

Québec

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Direction de la formation générale des jeunes

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Jean Garon
Minister of Education

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INTRODUCTION

Mathematics 314 is a program designed for Secondary III students attending schools in Québec.

To prepare young Quebecers for the demanding world of the twenty-first century, schools must emphasize the students' cognitive growth and the development of basic skills (i.e. communication and problem-solving skills as well as the ability to work with technology).

Because society is changing so rapidly, and owing to developments in the field of mathematics education, it is important to stress the interconnection of knowledge, skills and attitudes in the teaching of this program.

If the goal is to ensure that young people acquire a solid basic education, then mathematics education provides fertile ground for the development of the skills they will require in the future. As Resnick and Kloffer have noted, "Graduates must not only be literate; they must also be competent thinkers."[1]

1.1 Two Major Guiding Principles

Current knowledge of the learning process and the focus of student learning have led to an emphasis on two principles intended to guide teachers in their work with students. These principles are as follows: to encourage the students to participate actively in the learning process and to encourage them to use a problem-solving approach at each stage of the learning process.

Encouraging Students to Take an Active Part in Their Own Learning Process

A great many research studies have shown that students should play a central role in their own learning process. In short, they should be ultimately responsible for their education:

1. Lauren B. Resnick and Leopold E. Klopfer, "Toward the Thinking Curriculum: An Overview," in *Toward The Thinking Curriculum: Current Cognitive Research, 1989 Yearbook of the Association for Supervision and Curriculum Development*, ed. Lauren B. Resnick and Leopold E. Klopfer (Alexandria, Va.: Association for Supervision and Curriculum Development, 1989), 1.

The construction of a given concept is a complex process that depends first and foremost on the student. Concepts are not directly transmitted from a knowledgeable person to a student who supposedly knows nothing in a given field. Before they tackle new subject matter, students have already developed their own conceptions, which are well organized, practical and sometimes fairly resistant to the changes targeted in a course of study.

Thus, teaching involves creating situations in which students draw on their own knowledge. Teaching involves structuring the learning process around their strategies and thinking in order to try to get them to make progress in the construction of a given concept.[2]

To help students acquire the knowledge and skills targeted by this program, it is important to design learning situations that call upon their powers of observation and dexterity and that involve manipulation, exploration, construction and simulations. Through these activities, the students analyze hypotheses, actively look for solutions, discuss their approaches, analyze concepts or theories from their own point of view while taking into account other points of view, actively question the meaning and consequences of the procedures they use and relate the knowledge they have acquired to their own experience. These situations encourage the students to reflect, act, react and establish links with what they have already learned.

Another way teachers can encourage students to participate in their learning process is by developing a suitable teaching approach. Teachers will do more to help young people build their knowledge by asking them questions than by giving them the answers.

Any question that helps students get on the right track or find the answers to their questions by themselves encourages them to participate in their own learning.

2. Nadine Bednarz, "L'enseignement des mathématiques et le Québec de l'an 2000," excerpted from Richard Pallascio, ed., *Mathématiquement vôtre!, Défis et perspectives pour l'enseignement des mathématiques*, (Montréal: Les éditions Agence d'ARC inc., 1990), 69, (free translation).

Using Problem Solving at All Stages of Learning

Problem solving is an essential teaching and learning tool in several general education programs (e.g. pure sciences, social studies) and is an integral part of any mathematical activity. Problem solving is not a separate theme, but rather a process that should be applied throughout the program and that provides a suitable context for learning concepts and acquiring skills.

> Problem solving is both a basic skill that students should develop and an effective teaching approach that promotes the development of mathematical knowledge, thinking skills, socio-affective attitudes and problem-solving strategies.[3]

This approach calls for the active involvement of students and the use of questions. The teacher may ask the students questions, the students may ask themselves questions, or the students may ask one another questions.

To avoid any ambiguity, let us consider two categories of problems:

> [The first] category consists of problems with solutions requiring students to choose an appropriate combination of knowledge or skills from among several combinations seen in the past.... [The second] category consists of problems requiring students to create a new combination of knowledge and skills, exercise a great deal of intellectual independence and use plausible reasoning in order to solve them.[4]

In practice, most of the problems that students must solve fall between these two extremes. In order to achieve the terminal objectives, students must, at the very least, be able to solve problems belonging to category 1. However, the curriculum must go beyond this minimum standard if the goal is to create a program that truly reflects a problem-solving approach to mathematics education. The more exposure students have to problems (category 1 or 2) requiring them to associate a situation with a model, the easier it will be for them to analyze these situations and find a solution. Educators know that the ability to solve such problems presupposes the development of numerous high-level skills and thus necessitates that the learning process include many problem-solving opportunities.

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3. Québec, ministère de l'Éducation, *Mathematics Curriculum Guide, Elementary School, Booklet K, Problem Solving*, Code 16-2300-11A, (Québec: ministère de l'Éducation, 1989), 47-51.

4. Ibid, 15.

The problems can be related to the students' environment and used at various stages in the learning process. Problem solving may be used to introduce students to a new concept or skill, to help them develop this knowledge or ability or to help them review what they have learned.

Thus, problems provide an opportunity to:

— apply and integrate mathematical knowledge (e.g. concepts, properties, algorithms, techniques, procedures);

— develop intellectual skills (e.g. organizing, structuring, abstracting, analyzing, synthesizing, estimating, generalizing, deducing, justifying);

— develop positive attitudes (e.g. becoming aware of one's potential, respecting the opinions of others, and being imaginative and creative as well as rigorous and precise);

— use different problem-solving strategies (e.g. looking for patterns, representing a problem by means of a figure or a graph, constructing a table, referring to a known model, using a formula, formulating an equation, working backwards).

The emphasis on problem solving does not mean that exercises have no part in the teaching or in the learning of mathematics. Exercises play a different role, but one that is complementary to that of problem solving. For instance, they can be used to consolidate skills and habits to which the students have already been introduced, or to practise applying definitions and properties that the students have learned in class. Exercises can neither replace nor be replaced by problems.

If teachers take a problem-solving approach, the students become accustomed to referring to a known mathematical model and are thereby more likely to attain the terminal objectives. This approach can also assist the students in developing a procedure that will enable them to construct more knowledge and more models, thereby helping them attain the global objectives in accordance with the first guiding principle, namely, to encourage the students' active participation.

Students must have the opportunity to analyze their work methods and organize their thinking. In short, they must be able to learn how to learn.

1.2 Connection with Previous Programs

If there is continuity in learning, students can review the concepts they have already studied and develop their conceptions and representations. This mathematics program enables students to build on the knowledge acquired in elementary school and in the first two years of secondary school.

The learning process will be dynamic if the learning activities allow the students to use their knowledge and skills in new situations and help them become more proficient at applying what they have learned.

As they acquire new knowledge, the students will review the following skills and concepts acquired in previous programs:

— number and operation sense;
— the habit of estimating;
— sense of proportionality;
— understanding of the concept of a variable;
— translation from one mode of representation to another;
— definitions, properties, theorems or corollaries related to different geometric concepts;
— organization of statistical data;
— simulation of random events and the concept of probability.

Moreover, it should be noted that in the Secondary III program, arithmetic content has been incorporated into the terminal objectives relating to algebra, geometry and statistics.

1.3 Evaluation of Learning

Orientations and Practices Relating to the Evaluation of Learning

> The evaluation of student learning has come in for a great deal of discussion in the Québec education system over the last decade and it is surely no exaggeration to say that this field has been and to some extent remains a subject of scrutiny. Teachers today are more knowledgeable about the evaluation of student learning than they were in the past...[5].

It is important to draw on all the available expertise in evaluation and ensure that evaluation practices increasingly tie in with the essential learning pursued in the programs of study. Thus, the aim should be to establish greater coherence between the spirit of these programs and evaluation practices.

Procedures for Evaluating Learning

When evaluating student learning, teachers should keep in mind the purpose of evaluation. Whether the goal is to give immediate educational feedback (formative evaluation) or to determine whether one or more terminal objectives have been attained (summative evaluation), evaluation provides individual students with useful information about their learning progress. It also helps teachers to assess the organization of program content and the effectiveness of teaching methods. Since the program is aimed at helping students acquire a solid basic education and the skills that will enable them to adapt to a constantly changing society,

> ...the evaluation of learning should take into account the various components of human development and the complex nature of education, [and] be consistent with the learning activities carried out in the classroom...[6].

In this program, the students not only acquire knowledge, but also learn how to investigate, communicate, represent, reason and use a variety of approaches in order to solve problems. They also acquire other skills and attitudes.

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5. Conseil supérieur de l'éducation, *Évaluer les apprentissages au primaire : un équilibre à trouver* (Québec: Direction des communications du CSE, 1992), 1, (free translation).

6. Ibid., 2, (free translation).

Because the students' knowledge, skills and attitudes are constantly evolving it is necessary to create situations which will yield information that, after criterion- or norm-referenced interpretation, is likely to provide a reliable indication of the students' individual or collective knowledge.

Since "paper-and-pencil" evaluation may not be appropriate to every aspect of this program, a certain amount of adaptation will be necessary. Depending on the specific goals and in keeping with a spirit of diversification, the following evaluation procedures could be appropriate:

- Log
- Oral presentation of a solution or a mathematical subject
- Quiz
- Class discussion
- Group project
- Interview
- Comprehensive examination comprising a number of sections
- Evaluation during computer-assisted learning activities
- Observation checklist
- Self-evaluation
- ...

The different types of evaluation must also take into account the variety of learning activities:

- Manipulation activity
- Communication activity (oral or written, individual or group)
- Estimation activity
- Activity using a calculator
- Activity using a computer
- ...

The evaluation of learning, be it formative or summative, is essentially aimed at improving both learning and teaching.

As Esther Paradis notes in *L'évaluation des apprentissages : valoriser sa mission pédagogique*, it is essentially a matter of rediscovering the educational merit of evaluation.[7]

7. Esther Paradis, *L'évaluation des apprentissages : valoriser sa mission pédagogique*, (Québec: Fédération des enseignantes et enseignants des commissions scolaires, Centrale de l'enseignment du Québec, 1992), 26, (free translation).

1.4 Relative Importance of the General Objectives

The following table shows the relative importance of each general objective.

General Objectives	%
1. To help students learn to use algebra to generalize situations.	45
2. To enable students to apply their knowledge of geometric figures.	40
3. To help students develop the ability to analyze statistical data.	15

PROGRAM CONTENT

2.1 Program Structure

This program is made up of global, general, terminal and intermediate objectives. These objectives should reflect the aims of mathematics education and the principles put forward with respect to learning contexts.

Global Objectives:

Objectives that summarize the **role** that mathematics plays in providing students with the basic education they need to integrate into our changing society. These global objectives remain the same throughout the five years of secondary school and form the nucleus around which the objectives for each level are structured.

General Objectives:

Objectives that specify the context in which the global objectives will be pursued and that describe in general terms the **expected educational outcomes** associated with each program theme. General objectives can be broken down into a set of terminal objectives.

Terminal Objectives:

Objectives that clarify the general objectives and describe the **anticipated results**. Each terminal objective is described in three paragraphs:

- The first paragraph indicates what the students learned in *Mathematics 116* or *Mathematics 216*, as the case may be.

- The second paragraph provides criteria for determining whether the students have attained the terminal objective.

- The third paragraph outlines activities that are consistent with the general objective, the global objectives and the educational principles. In this way, it reflects the spirit of the program.

The terminal objective is attained when the students are able to establish a link between a situation and acquired knowledge. This ability is directly related to attainment of the terminal objective and not to attainment of each of the underlying intermediate objectives, a complex object of knowledge being more than the sum of its parts. Hence, the goal is to have the students achieve the terminal objectives of the program. The degree to which the terminal objectives of the program are attained is directly related to the appropriateness of the measurement instruments, which must take into account the scope of the intermediate objectives and the context outlined by the general objective and the global objectives.

Intermediate Objectives:

Objectives that specify the scope of a terminal objective, intermediate objectives might also be described as "reference objectives." They are not intended as a series of steps to be completed one after the other. Such a process would give a very fragmented picture of teaching and learning. Rather, intermediate objectives are:

- aspects of a theme that have been chosen for the program;
- clarifications to ensure that the terminal objective is clearly understood;
- guidelines that indicate the connection between the terminal objective and student learning;
- prerequisites for attaining a terminal objective.

2.2 Program Objectives

Global Objectives

Establishing Connections

Increasing the students' ability to establish connections between the knowledge they are acquiring and the knowledge they already have in mathematics and other disciplines, and encouraging them to view their knowledge as a tool that can be useful to them in everyday life.

Problem Solving

Increasing the students' ability to analyze the data associated with a problem and use appropriate strategies to arrive at a solution that they will be able to verify, interpret and generalize.

Communicating

Increasing the students' ability to grasp and transmit information and to express their thoughts clearly, using mathematical language.

Reasoning

Increasing the students' ability to formulate hypotheses and verify them using an inductive or a deductive method.

General Objective 1

To help students learn to use algebra to generalize situations

Since we are now living in the information age, students should be equipped to handle, process and interpret the information they will encounter.

In Secondary II, students learned that algebra was a powerful and useful language or communication tool. They were introduced to different modes of representation (see table under Objective 1.1) which highlighted certain aspects of problems they had to solve. In Secondary III, students will expand their knowledge in this area by using algebra to derive general rules from a number of specific situations. Conversely, they will apply these general rules to individual cases.

Using simple experiments or situations which may have been suggested by the students, the teacher will help them discover the type of dependence characterizing the relationship between certain variables. The students will be able to relate the characteristics of various modes of representation to the situation they are examining. Among other things, they will realize that these situations are not always represented by a straight line. They will continue their investigation by giving a more detailed description of situations illustrated by straight lines.

Like any other language, algebra has its own rules and syntax, and it is important that the students observe them. After performing operations on arithmetic expressions given in exponential form, they will generalize the properties of these operations and then apply these properties to algebraic expressions. In so doing, the students continue learning about algebraic manipulations. In this case, literal expressions do not have to represent a specific value and are used as mathematical objects.

The Pythagorean theorem will illustrate the link between algebra and geometry, since the students will be able to solve problems using the algebraic generalization of a property of right triangles. This theorem will even help them understand the concept of irrational numbers, thereby allowing them to complete their study of the set of real numbers.

Terminal Objective 1.1

To illustrate the type of dependence characterizing the relationship between the variables in a situation

In Secondary II, the students used different modes of representation to describe and represent a situation. They also developed their ability to reason proportionally.

Students who have attained Terminal Objective 1.1 of this program will be able to use different modes of representation to analyze a situation so that they can then express the relationship between the variables in their own words. The students will be given situations they can understand and in which the variables are directly or inversely proportional or in which one of the variables is proportional to the square of the other. They must understand that the relationship between the variables is not always linear and informally identify the characteristics of the different types of dependence. The students are not expected to be able to write an equation for a given situation or to identify the type of relation. The shaded boxes in the following table indicate the translations covered in *Mathematics 314* and the boxes marked with an asterisk indicate the translations studied in *Mathematics 216*.

TRANSLATIONS FROM ONE MODE OF REPRESENTATION TO ANOTHER

to / from	words or drawing	table of values	graph	rule or equation
words or drawing	*	*	*	
table of values	*			
graph	*			
rule or equation				

Situations that give rise to discussions and questions that involve analyzing the relationship between variables are consistent with the global objectives, General Objective 1 and the guiding principles. The students will develop their powers of observation and their ability to analyze and synthesize a situation.

1.1

Intermediate Objectives

- To determine the dependent variable and the independent variable in a given situation.
- To represent the rule that applies in a given situation, using a table of values.
- To represent a situation and its corresponding rule by means of a graph, given a table of values.
- To express in their own words the relationship between the variables in a specific situation, given the description of that situation, a table of values or a graph.

Terminal Objective 1.2

To solve problems related to situations in which a linear relationship exists between the variables

In Secondary II, the students acquired certain skills that enabled them to write a first-degree equation for a given situation. In studying ratios and proportions as well as the material covered in Objective 1.1 of this program, they explored situations involving direct variation.

Students who have attained Terminal Objective 1.2 of this program will be able to solve problems involving direct or partial variation. In a situation involving direct variation, the dependent variable is expressed as a single term consisting of the product of the independent variable and a constant. In a situation involving partial variation, the dependent variable is expressed as the sum of two terms, one being the product of the independent variable and a constant and the other being a constant. The objective here is not to start a debate about the meaning of "situation involving direct variation" and "situation involving partial variation," but rather to have students examine situations represented by a straight line. The students will analyze these situations in greater detail by associating each type of variation with a specific form of equation, describing the role of the rate of change and determining how a parameter change will affect the situation, the graph and the equation. The students must learn that straight lines are sometimes used in situations involving only integers, or that only a part of a line is significant. They will also study situations in which the rate of change is constant within a given interval and then assigned a different value within another interval, as is the case with broken-line graphs. Functional notation should not be used, because it is important that students be able to observe and explore situations without being distracted by overly complex symbolism. The shaded boxes in the following table indicate the scope of this objective.

TRANSLATIONS FROM ONE MODE OF REPRESENTATION TO ANOTHER

to from	words or drawing	table of values	graph	rule or equation
words or drawing				
table of values				
graph				
rule or equation				

Activities in which the students learn how to interpret graphs and to understand the relationships between symbolic, graphic and numerical representations are consistent with the global objectives, General Objective 1 and the guiding principles. It would be useful to employ a variety of learning aids and methods (i.e. mental calculations, "paper-and-pencil" exercises, graphic display calculators, computers).

1.2

Intermediate Objectives

- To translate a situation involving direct variation or partial variation into an equation.
- To translate an equation involving direct variation or partial variation into a word problem.
- To determine the rate of change in a situation involving direct variation or partial variation, given the corresponding equation or graph.
- To provide a qualitative description of how a parameter change will affect a graph, given the equation for a situation involving direct variation or partial variation.

Terminal Objective 1.3

To convert an arithmetic or algebraic expression into an equivalent expression

In the Secondary I program, the students gradually developed their understanding of the four operations and their ability to perform these operations on rational numbers. In Secondary II, they added and subtracted expressions containing one variable and constants and multiplied and divided this type of expression by a constant.

Students who have attained Terminal Objective 1.3 of this program wil be able to convert an arithmetic or algebraic expression into an equivalent expression. They must be able to perform operations on numerical and algebraic expressions containing exponents. They must not only find the simplest expression, but also recognize and give other equivalent expressions. Appropriate visual aids may be useful in introducing the relevant algebraic manipulations.

Activities in which the the students gradually expand their knowledge of the structure of algebra while developing their ability to perform certain operations are consistent with the global objectives, General Objective 1 and the guiding principles. The focus should be on the students' understanding of these operations rather than on the mechanics of performing them. The students will also be able to apply the skills they have developed with respect to number and operation sense. By establishing the properties of exponents, the students continue to learn how algebra can be used to generalize situations.

1.3

Intermediate Objectives

- To apply the properties of exponents in transforming arithmetic expressions.
- To apply the following properties in transforming algebraic expressions:

$$n^a \cdot n^b = n^{a+b} \text{ where } a, b \in \mathrm{N}$$

$$\frac{n^a}{n^b} = n^{a-b} \text{ where } a, b \in \mathrm{N}$$

- To add and subtract polynomials.
- To multiply a monomial by a polynomial and a binomial by a binomial.
- To divide a polynomial by a monomial.

Terminal Objective 1.4

To solve problems using the Pythagorean theorem

In Secondary I and II, the students learned how to calculate the measure of one of the dimensions of a polygon given sufficient information, while assimilating the concept of square root. In studying the material covered in Terminal Objectives 1.1, 1.2 and 1.3 of this program, they used algebra to generalize situations.

Students who have attained Terminal Objective 1.4 will be able to solve problems in which they can apply the Pythagorean theorem. The goal is to ensure that the students can state this property using words and algebraic symbols. They will also use the converse of the Pythagorean relation to establish whether a triangle is right-angled. They will also learn about irrational numbers and locate them on the number line using a ruler and compass.

Construction, exploration, observation and discussion activities in which students can derive the Pythagorean theorem and use it to solve a variety of problems are consistent with the the global objectives, General Objective 1 and the guiding principles. The students continue to learn how algebra can be used to generalize situations. They will also see that this property of right triangles is applied in various areas of human activity.

1.4

Intermediate Objectives

- To express the Pythagorean relationship between the measures of the sides of a right triangle, using variables.
- To justify an assertion used in solving a problem that involves applying the Pythagorean theorem.*
- To locate some irrational numbers on a number line.

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* See Appendix.

General Objective 2

To enable students to apply their knowledge of geometric figures

Geometry helps us represent and describe our world. In Secondary III the students continue to develop their perception of space in the world around them.

The students progress through a hierarchy of levels in developing their geometric thinking skills. They first learn to recognize shapes and then analyze the different properties of these shapes before establishing relationships between the properties and making simple deductions. Through various active exploration and observation activities, the students establish a system of relationships pertaining to triangles, quadrilaterals, circles and regular polygons. In the Secondary III program, this system will be expanded to include solids.

The students live in a three-dimensional world which they began to understand through orientation and observation activities in elementary school. In Secondary III, it is essential to continue developing the students' spatial sense, which is a mental skill that makes it possible to create and manipulate the images of objects. Hence, in determining the result of a composite of transformations, the students must try to imagine the one transformation that produces the same outcome. The goal is to improve the students' perception of two- and three-dimensional space, of the objects in that space and of the qualitative and quantitative aspects of their own representations.

Terminal Objective 2.1

To solve problems involving isometries or dilatations

In Secondary I and II, the students constructed figures resulting from isometries or dilatations and explored the properties of these transformations. They also constructed these figures in a Cartesian plane.

Students who have attained Terminal Objective 2.1 will be able to use their knowledge of composite transformations to solve problems. The students will begin by constructing composite transformations and assimilating the concept of an inverse transformation. They will then analyze the situation and, if possible, identify the one transformation that is equivalent. After examining a sufficient number of transformations, the students will be able to state the main properties of each type of transformation. They can study these transformations in a plane using a geometry set or analyze them in a Cartesian plane. In the latter case, the limitations established in Secondary II also apply (i.e. reflections with respect to the axes or the bisectors of the quadrants, rotations centred at the origin with the rotation angle being a multiple of 90° and dilatations centred at the origin).

Activities in which the students study transformations of the plane in greater detail by identifying their properties are consistent with the global objectives, General Objective 2 and the guiding principles. Following observation and construction activities, the students will be able to use the correct terminology to state these properties. Visual aids are extremely important in geometry because they help support one's reasoning. Working in groups, the students can use different types of aids (e.g. transparencies, computer simulations) to analyze different situations and arrive at a consensus.

2.1

Intermediate Objectives

- To construct the image of a figure under a composite of transformations.
- To describe the inverse of different transformations of the plane.
- To identify the transformation that is equivalent to a given composite of transformations.
- To state the main properties of the different transformations.*

* See Appendix.

Terminal Objective 2.2

To solve problems involving three-dimensional objects

In elementary school, the students established spatial relationships between objects by locating them or by making networks, frieze patterns or grids.

Students who have attained Terminal Objective 2.2 of this program will be able to perceive three-dimensional objects and represent them in different ways. Given a set of cubes, students must give a verbal description of what they see, draw sketches of the front, side and top views or describe the various layers of a given object. An introduction to basic techniques such as the Cavalierian perspective or the use of isometric dot paper will help the students produce better representations of three-dimensional objects. The students will also be asked to construct three-dimensional objects on the basis of descriptions or two-dimensional representations.

Activities that help the students better understand spatial relations are consistent with the global objectives, General Objective 2 and the guiding principles. In these activities, students should alternate between manipulations, the examination and production of drawings, mental exercises, abstractions and deductions. While helping students expand their knowledge of concepts, the teacher should also help them learn how to visualize space and represent solids. An emphasis on group work will give the students the opportunity to compare their points of view as they formulate and test hypotheses.

2.2

Intermediate Objectives

- To describe three-dimensional objects, using words or drawings.
- To represent three-dimensional objects in two dimensions.
- To build a three-dimensional object on the basis of a description or a drawing.

Terminal Objective 2.3

To solve problems involving solids

In the first cycle of elementary school, the students discovered certain characteristics of solids. In the second cycle, they explored certain ways of arranging plane figures to construct solids and identified the characteristics of these solids. In Secondary I, they created plane figures through isometric transformations.

Students who have attained Terminal Objective 2.3 of this program will have consolidated their knowledge of solids in order to solve a variety of problems. They will create solids through rotations or translations of plane figures or by splitting a cube into sections. They will analyze the elements of solids (e.g. edges, sides, vertices, diagonals, altitudes, slant heights (apothems)) and use these elements to classify solids. The students must also be able to deduce a measure in a solid or in a figure they have already created.

Activities in which the students assimilate the terminology relating to solids, learn how to construct them, identify the properties of a given solid and establish the connections that make it possible to solve problems are consistent with the global objective, General Objective 2 and the guiding principles. The students need a tool that will help them to improve their spatial perception and to develop the mental images which can be used to support their reasoning. They will continue to justify their reasoning using relevant definitions and properties, including those studied in Secondary I and II.

2.3

Intermediate Objectives

- To generate a cone, sphere or cylinder by rotating a figure 360° about an axis.
- To generate a prism by translating a polygon.
- To represent solids in two or three dimensions.
- To classify solids.
- To split a cube into sections to obtain a solid with one triangular or quadrilateral face.
- To deduce the measure of a segment from an appropriate definition or property.*
- To justify an assertion used in solving a problem involving solids.*

* See Appendix.

Terminal Objective 2.4

To solve problems related to the area or volume of certain solids

In elementary school, the students estimated and measured volume and even calculated the volume of certain simple solids. In the first two years of secondary school, they calculated the perimeter and area of the following plane figures: triangles, quadrilaterals, circles and polygons.

Students who have attained Terminal Objective 2.4 of this program will be able to establish relationships between the dimensions of a solid and its area or volume in order to solve problems. The different principles of geometry studied earlier in this course will be incorporated into the activities related to this objective. These activities should also allow students to consolidate their knowledge of the concepts of area and volume, establish connections between the characteristics of solids and their measures, use the appropriate units of measure and apply their knowledge to solids that can be broken up into several parts.

Activities in which the students learn how to apply the properties of solids in order to find their area and volume are consistent with the global objectives, General Objective 2 and the guiding principles. By studying the measures of solids, the students are in fact continuing to analyze these figures. By establishing the formulas for calculating the volume of various solids, the students continue to learn how algebra can be used to generalize situations.

2.4

Intermediate Objectives

- To distinguish between situations in which the area should be calculated and those in which the volume should be calculated.
- To calculate the area of solids that can be broken up into right solids or hemispheres.
- To express the relationship between the volume of a solid and some of its measures.
- To calculate the volume of objects that can be broken up into right solids or hemispheres.
- To convert a measure of volume from one unit to another.
- To establish the connection between units of volume and units of capacity.
- To calculate a measure of a solid, given its area and sufficient data.
- To calculate a measure of a solid, given its volume and sufficient data.
- To justify an assertion used in solving a problem involving solids.*

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* See Appendix.

General Objective 3

To help students develop the ability to analyze statistical data

We live in an age dominated by information and technology. The ability to understand and make intelligent use of information in all its forms has become increasingly important in all types of industrial, commercial and governmental organizations. Students often encounter data presented and interpreted in a variety of ways. A knowledge of descriptive statistics can help them pinpoint the essential facts about a situation that are concealed in a mass of data. The students will then be better equipped to understand situations and assess them critically.

In Secondary I, the students learned how to present data in tables or graphs so as to highlight various aspects of a situation. They continue to develop these skills in Secondary III by constructing histograms that illustrate a distribution of continuous quantitative data. The graph of a distribution provides a brief description of the data in question. The students will acquire other tools for analyzing data by learning how to derive information from measures of central tendency and the range of a distribution.

Statistics provides an excellent opportunity to study simple, concrete situations that show the connections between mathematics and the real world. Students should be encouraged to examine various aspects of a situation, to structure and organize data, and to formulate and test hypotheses. In this way, they will develop their inductive-reasoning ability while playing an active role in their own education. The use of computers and calculators will allow them to focus on the interpretation of measurements rather than on calculation methods.

Terminal Objective 3.1

To solve problems involving situations represented by a one-variable statistical distribution

In Secondary I, the students learned how to use tables and graphs to present information about a situation.

Students who have attained Terminal Objective 3.1 will be able to solve problems by using certain methods for analyzing statistics and interpreting data. While becoming familiar with the relevant terminology, the students will be asked to present data in the form of tables or histograms. They will use measures of central tendency to analyze or describe a distribution.

Activities involving more comprehensive learning contexts in which the students can apply their knowledge and engage in a constructive exchange of ideas are consistent with the global objectives, General Objective 3 and the guiding principles. The students will learn to approach situations as a statistician would—by thinking critically, analytically and logically about the problem at hand.

3.1

Intermediate Objectives

- To tabulate data.
- To present data in the form of a histogram.
- To calculate the mean, median, mode and range of a distribution consisting of data that has not been grouped into classes.
- To derive qualitative information about a distribution from its mean, median, mode or range.
- To describe a distribution, given its mean, median, mode or range.

APPENDIX

Statements Associated with Themes Covered in *Mathematics 314*

While studying geometry in this course, the students examine the Pythagorean theorem, geometric transformations and solids. Through learning activities, they increase their grasp of a number of concepts and improve their skills. They should also become familiar with the definitions of the figures they are studying, with some of their properties (see below) and with the properties of geometric transformations. The students will use these definitions and properties of figures to calculate measurements and justify any assertions used in solving problems involving solids.

1. In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other sides.

2. A triangle is right-angled if the square of the length of one of its sides is equal to the sum of the squares of the lengths of the other sides.

3. In any convex polyhedron, the sum of the number of vertices and the number of faces is equal to the number of edges plus two.

4. Isometries or dilatations have one or more of the following properties:

   - they preserve collinearity;
   - they preserve parallelism;
   - they preserve the order of points;
   - they preserve the orientation of the plane;
   - they preserve distances and the measures of angles.

5. Any translation and any dilatation will transform a straight line into another line parallel to it.

In the Secondary I and Secondary II programs, the students began to build up a system of axioms. In order to deduce certain measurements and justify certain steps involved in solving problems, the students must apply the following statements as well as those studied in Secondary III.

Statements Studied in *Mathematics 116*

1. Adjacent angles whose external sides are in a straight line are supplementary.
2. Vertical angles are congruent.
3. The sum of the measures of the interior angles of a triangle is 180°.
4. In any triangle, the length of any side is less than the sum of the lengths of the other two sides.
5. In any triangle, the length of any side is greater than the difference of the lengths of the other two sides.
6. In any triangle, the longest side is opposite the largest angle.
7. In any isosceles triangle, the angles opposite the congruent sides are congruent.
8. In any equilateral triangle, each of the angles measures 60°.
9. In any right triangle, the acute angles are complementary.
10. In any isosceles right triangle, each of the acute angles measures 45°.
11. The axis of symmetry of an isosceles triangle contains a median, a perpendicular bisector, an angle bisector and an altitude of the triangle.
12. The axes of symmetry of an equilateral triangle contain the medians, perpendicular bisectors, angle bisectors and altitudes of the triangle.
13. The opposite angles of a parallelogram are congruent.
14. The opposite sides of a parallelogram are congruent.
15. The diagonals of a parallelogram bisect each other.
16. The diagonals of a rectangle are congruent.
17. The diagonals of a rhombus are perpendicular to each other.

Statements Studied in *Mathematics 216*

1. The diagonals from one vertex of a convex polygon form $n - 2$ triangles, where n is the number of sides in that polygon.

2. In a convex polygon, the sum of the measures of the exterior angles, one at each vertex, is 360°.

3. The sum of the measures of the interior angles of a polygon is $180°(n - 2)$, where n is the number of sides in the polygon.

4. Three non-collinear points determine one and only one circle.

5. All the perpendicular bisectors of the chords of a circle meet at the centre of that circle.

6. All the diameters of a circle are congruent.

7. In a circle, the measure of the radius is half the measure of the diameter.

8. The axes of symmetry of a circle contain its centre.

9. The ratio of the circumference of a circle to its diameter is a constant known as π.

10. In a circle, the measure of the central angle is equal to the measure of its intercepted arc.

11. In a circle, the ratio of the measures of two central angles is equal to the ratio of the measures of their intercepted arcs.

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