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Adaptive two-stage stochastic optimization of the Escondida mining complex, Chile

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Abstract: This paper presents the application of adaptive simultaneous stochastic optimization with a representative branching framework to generate the strategic plan of the Escondida mining complex, the world's largest copper-production operation. This adaptive, two-stage stochastic optimization considers geological uncertainty and integrates investment and operational alternatives in the production schedule. Mining complexes are comprised of interconnected components affected by multiple sources of uncertainty. Thus, they must be optimized simultaneously in order to maximize their value and manage risk. Additionally, due to the extensive lives of assets, it is not possible to assume that the current strategic plan will remain optimal. Thus, an operationally feasible method to embed alternatives in the mine plan is used. The method presented provides a strategic plan with representative branches for future possible investment decisions. Adaptive decisions are made sequentially over time, activating costs and effects over the model. The optimizer chooses the optimal strategic production plan accordingly, as well as the investments made and their timing. The Escondida mining complex is a multi-element, multi-pit operation with nine different processing destinations. Investment options considered are increasing truck and shovel fleet, adding a secondary crusher in one of the plants, and investing in a main crusher assigned to one of the pits. Additionally, operational alternatives at the mine and plant levels are included. The adaptive solution shows a substantial probability that the mine plan might change its design substantially due to geological uncertainty, presenting an increased expected NPV compared to the two-stage stochastic formulation.

1 Introduction

Escondida is the world's largest copper-producing mining complex, located at over 3,000 m.a.s.l. in the Antofagasta Region in northern Chile. It is operated by Minera Escondida Ltd. and is part of BHP's operations (Padilla et al., 2001). A mining complex consists of a set of connected, interdependent components, including a set of mines, which supply raw material, stockpiles, different processing streams, and a set of final destinations that sell the processed material for a profit. A diagram of the Escondida mining complex is presented in Figure 1, which consists of two open-pit mines, Escondida and Escondida Norte, both of which are part of the multi-element Escondida porphyry. There are four material types defined as sulfides, oxides, mixed, and waste. Sulfides can be processed by three different sulfide processing plants, which are fed by four crushers and receive material from both mines; low-grade sulfide material can also be sent directly to a bio-leach pad as run-of-mine (ROM). Oxide and mixed material must be processed in a separate leach-pad, which is fed from a fifth crusher, also receiving material from both mines. Additionally, there are two stockpiles available for oxide and sulfide material, and a waste dump, which are fed directly from the pits.

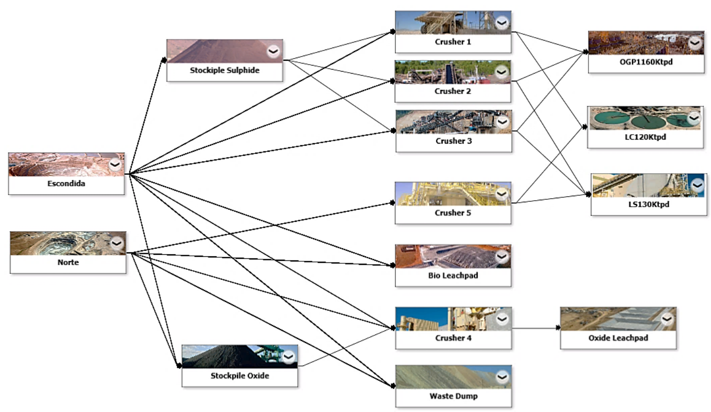

Figure 1: Diagram of the Escondida Mining Complex

Mining complexes are governed by inherent uncertainties both internally (geological, technical), as well as externally (royalties, markets, etc.) (Dowd, 1997, 1994; Johnson, 1968; Ravenscroft, 1992). In order to obtain a reliable mine plan, a mining complex must be optimized simultaneously, accounting for the value created by the synergies that exist between its components (Bodon et al., 2011; Hoerger et al., 1999; Pimentel et al., 2010; Whittle, 2010, 2007), as well as for the effects of uncertainty over its strategic plan (Goodfellow and Dimitrakopoulos, 2016; Kumar and Dimitrakopoulos, 2019; Lamghari et al., 2015; Montiel et al., 2016; Montiel and Dimitrakopoulos, 2018; Saliba and Dimitrakopoulos, 2018). Montiel and Dimitrakopoulos (2015) develop a model to optimize the production schedule of a mining complex that considers supply uncertainty, including operating alternatives at the processing plant and the transportation systems of the mineral value chain. To account for supply uncertainty, the authors use a set of stochastically simulated realizations of the deposit, which represent the local variability of the attributes of interest, such as the grade of metal, material ore types, or deleterious elements (Goovaerts, 1997). Similarly, Goodfellow and Dimitrakopoulos (2017) present a simultaneous stochastic optimization framework that integrates decisions over material extraction from a set of sources along with their uncertainty, as well as blending, stockpiling, processing and transportation decisions, while managing the related risk to meet production targets and maximize revenue. Goodfellow (2014) extends this model to include capital investment alternatives, allowing the optimizer to define extraction capacities as part of the optimization process. However, all of these studies provide static solutions, fixed for the whole LOM after optimization, and do not propose any feasible alternatives to adapt a strategic plan if the expected future changes, underestimating the asset's potential (Dowd et al., 2016; Eckart et al., 2010; Wang, 2005).

To deal with this lack of flexibility, Del Castillo and Dimitrakopoulos (2019) propose an adaptive, two-stage stochastic programming model with representative branching, which allows for the inclusion of investment alternatives into the strategic plan. This model provides a clear image of possible future evolutions of the asset, and develops straightforward implementation plans for them. The strategic plan produced is presented as a scenario tree, with branches defined through representative scenario groupings, which show all mine-design investment developments that have a representative probability of occurring given the uncertainty. In order to control the complexity of the model and limit the growth of the solution tree, only investments that have an important effect over the strategic plan are considered for branching. Thus, investment decisions are divided into two sets, branching ($\boldsymbol{K}^{<}$) and non-branching ($\boldsymbol{K}^{=}$). The former are major investments with extensive lead times, and are taken only once, or once every more than ten years in the LOM, such as the opening of a new processing plant. The later have a low relative impact over the LOM schedule, and/or are multiple small decisions taken repeatedly over the LOM, such as truck purchases, which define the mines' extraction capacity. The proposed approach provides a probabilistic analysis of investing or not in different big capital expenditures, with their corresponding investment timings, production plans, and extraction capacities.

Due to the long life of assets and the uncertainties affecting it, expecting that their setup will remain unchanged throughout its life is a strong assumption that might hinder the potential to maximize revenue. The Escondida mining complex has a reported life of asset of 58 years, as of January 2018 ("Mining Data Solutions - Escondida Mine," 2018), thus, it is necessary to develop a strategic plan that not only respects production forecasts, but that also enables change. Traditionally, mine plans are updated annually according to the previous year's information on costs, commodity price evolutions, and extracted material. However, this is a suboptimal practice that only allows reactive responses, inhibiting timely and efficient large-scale changes and investments, as these require extensive lead times and coordination between the components involved. The simultaneous stochastic optimization of a mining complex (Goodfellow and Dimitrakopoulos, 2017; Montiel and Dimitrakopoulos, 2015) manages risk and maximizes value given the mining complex's setup, but also produces static plans.

Project value can be maximized by generating strategic plans that react in a timely way to change (Siegel et al., 1987). For this, flexibility alternatives must be created and maintained within the model, optimizing the type and timing of new investments that could become valuable given future changes. A "flexible design" is defined as being able to adapt and reconfigure if needed (De Neufville et al., 2004; De Neufville and Scholtes, 2011). Accordingly, flexibility in strategic plans is often translated into a set of different possible solutions that are operationally impossible to follow in real life. For strategic plans and financial assessments to be reliable, the strategic plan and mining design must be operationally feasible. In mining terms, this means that they must follow all operational requirements and physical geotechnical restrictions. Including flexibility in mining operations has been a topic of interest in the technical literature, tackling different sources of uncertainty, such as commodity price, geology, or operating costs (Boland et al., 2008; Groeneveld and Topal, 2011; Kazakidis and Scoble, 2003; Singh and Skibniewski, 1991), but they have failed to produce optimized mine plans that allow for feasible, implementable designs.

Mining operational requirements demand a unique strategic plan for the life of assets, mostly in order to define clear medium- and short-term plans. However, the strategic plan may allow variations and alternatives during later periods without affecting short-term planning. The model developed by Del Castillo and Dimitrakopoulos (2019) extends past approaches on simultaneous stochastic optimization of mining complexes by including feasible implementations of alternatives in the strategic plan through the dynamic planning of investments, providing a broader look at possibly profitable evolutions of the plan. Del Castillo (2018) extends this work by including decisions over operational alternatives into the model, letting the optimizer define blast-hole patterns to affect rock fragmentation, as well as plants' throughput and recovery configurations. This work stresses the advantages of integrating the optimization of different investment and operating alternatives into one model, as a part of the strategic planning process, as well as the benefits of allowing a mine design to branch, planning for investment decisions that might be profitable in the future.

This paper outlines the application of the adaptive stochastic optimization model at the Esconida mining complex in the presence of supply uncertainty. For comparison, the mining complex is also optimized using the traditional two-stage stochastic simultaneous optimization, with and without considering investment and operating mode alternatives. Results show substantial benefits in considering dynamic alternatives within the optimization, both in terms of net present value (NPV) as well as the use of available equipment. The next section briefly describes the adaptive methodology. Section 3 presents the application at the Escondida mining complex and introduces the different cases compared. Conclusions follow.

2 An adaptive simultaneous optimization method

The adaptive, two-stage stochastic programming model with representative branching proposed by Del Castillo and Dimitrakopoulos (2019) aims at maximizing value while managing risk due to the presence of uncertainties. In this case, uncertainty in supply is represented through a set of S geological simulations of the deposit, and the strategic plan is optimized over a period of T years, aiming at maximizing the following objective function.

$$\max \frac{1}{|S|} \sum_{s \in S} \sum_{t \in T} \left(\text{Discounted Profit}_{s,t} - \text{Investment Costs}_{s,t} - \text{Penalty for Deviations}_{s,t} \right) \tag{1}$$

The first term of Eq. 1 corresponds to all discounted revenues from the final products, minus extraction and processing costs. The second term considers directly the purchase cost of the different investments acquired along the LOM, also discounted to present value. Finally, the third term aims at managing risk by minimizing deviations from production targets. These targets consider maximum production and extraction capacities, as well as blending targets and constraints in the different processing streams.

To allow the adaptive two-stage optimization model to branch, ensuring that decision variables remain constant within a branch, and differ only between separate branches, non-anticipativity constraints are included (Birge and Louveaux, 1997). These non-anticipativity constraints are the only constraints linking the separate scenarios, and ensure that decisions are *nonanticipative* of future outcomes, and in this case, are present for all extraction sequence decisions, destination policy decisions, operating mode decisions, and non-branching investment decisions. A representation of the set of constraints related to the extraction sequence decisions ($x_{b,t,s}$) is presented in Eq. (2), where $x_{b,t,s}$ equals to 1 if block b of mine M is extracted in period t, in scenario s, and 0 otherwise.

$$(1 - A_{t-1}) \left(x_{b,t,s} - x_{b,t,s'} \right) = 0, \qquad \forall t, t-1 \in T;\ b \in M;\ s \in \Omega_{\rho 1};\ \ s' \in \Omega_{\rho 2};\ \ \Omega_{\rho 1,2} \in \Omega_{\rho} \tag{2}$$

Given that Ω_ρ is the set of scenarios in that branch, $\Omega_{\rho 1} \bigcup \Omega_{\rho 2} = \Omega_\rho$ are scenario partitions, where $\Omega_{\rho 1} = \{s\ ;\ inv. = \text{true}, \forall s \in \Omega_\rho\}$, $\Omega_{\rho 2} = \{s\ ;\ inv. = \text{false}, \forall s \in \Omega_\rho\}$. Variable A_t is *activated* (equals to 1) in a given period t if there is a representative probability R^* of investing and not investing in a branching capital expenditure, eliminating the constraint, and thus, allowing decisions to vary for the following planning period (t). However, if A_t is not activated (equals to 0), then constraint 2 enforces all extraction decision variables to be equal throughout all scenarios (see Del Castillo (2018) for details on the calculation of A_t and for the full model). The actual decision to branch is taken by calculating the representativity of the probability of purchasing a branching investment. For this, a threshold parameter $R \in [0, 0.5]$ is defined, where branching only occurs when the probability of investing (R^*) falls within this threshold ($\in [R,\ 1-R]$). If the probability of investing is lower than the threshold ($R^* \in [0, R)$), the solution does not branch, and no investment is made. On the other hand, if the probability is higher than the threshold ($R^* \in (1-R, 1]$), there is also no branching, but the full mine plan invests.

The proposed adaptive, two-stage stochastic programming model is solved by using a rolling-horizon decision-making mechanism (Adulyasak et al., 2015; Bertsekas et al., 1997; Sethi and Sorger, 1991), which iteratively fixes decisions on an increasing time horizon, and allows later periods to differ. This process quantifies investing probabilities and, if these probabilities are representative, branches and rolls back to generate feasible strategic plans for each branch, later fixing the decisions taken until that period. This process is repeated until all periods of the LOM are fixed.

3 Application at the Escondida mining complex

3.1 Overview

As mentioned previously, the Escondida Mining Complex (presented in Figure 1) consists of two open-pit mines, Escondida and Norte, which contain over 120 and 78 thousand blocks, respectively. Each block measures 25m x 25m x 15m in dimension, and contains a variable concentration of copper, gold, silver, and molybdenum as valuable elements, as well as arsenic and iron which must be controlled. The mines are connected to four crushers that feed three sulfide processing plants, OGP1, LC, and LS, which have a processing capacity of 160, 120, and 130 thousand tonnes per day (ktpd) respectively, as well as a crusher that feeds an oxide leach-pad with a capacity of 25 million tonnes per year. Material from both mines can also be sent to a bio-leach pad that has the capacity to treat 135 million tons of run of mine (ROM) material per year, and a waste dump with assumed infinite capacity. Additionally, there are two stockpiles available for oxide and sulfide material, which are fed directly from the pits and have a capacity of 75 and 30 million tons per year respectively.

Exploration drill-holes show that the life of mine is in the order of decades, however the strategic plan is defined for a time-range of 8 years. The main product sold by the mining complex is copper concentrate produced by the processing plants, which has a premium for gold, silver, and molibdenum content. Copper cathodes are also produced by the oxide and bio-leach pads. Escondida has an initial fleet capacity of 98 trucks and 14 shovels, and Norte has 42 trucks and 6 shovels assigned for the first two years of the strategic plan. Extraction capacity after that point will be defined by the optimizer for both mines, by investing in trucks and shovels. Because of geotechnical constraints, Escondida can have a fleet of up to 120 trucks, and Norte of up to 70. Due to cycle times and maintenance, for optimal performance, it is considered that a shovel can haul up to 7 trucks.

3.2 Alternatives considered

The alternatives included into the Escondida Mining Complex are divided into operational and investment alternatives. Operational alternatives correspond to intrinsic flexibilities that allow for adapting the configuration of a process at a given component of the mining complex, and investment alternatives are capital expenditures in equipment or infrastructure that are feasible to consider within the mine plan.

3.2.1 Investment alternatives

A summary of the investment alternatives considered is presented in Table 1. Here, the two sets of investments defined earlier are presented, which are periodic investments ($\boldsymbol{K}^{=}$) and one-time investments ($\boldsymbol{K}^{<}$). In this case, the first set includes truck and shovel purchases for the Escondida and Norte pits separately, which jointly define the extraction capacity of the mining complex, and the second considers investments over a secondary crusher at the LS plant, and an extra crusher assigned to the Escondida pit, which feeds material to OGP1 and LS plants.

3.2.2 Operational alternatives

In this case, as in Del Castillo (2018), two sets of operational alternatives are considered. These are i) adapting the processing modes at each of the three plants, increasing the metallurgical recovery, but at a cost in throughput, and ii) applying a change in the blasting pattern at each of the mines, in order to help fragmentation, but at a higher mining cost due to extra explosives and blasting perforations.

Table 1: Parameters of the investment alternatives considered in the mining complex

Equipment Parameters	Truck ($K^{=}$)	Shovel ($K^{=}$)	2ry Crusher in LS ($K^{<}$)	Extra Main Crusher ($K^{<}$)
Undiscounted cost	MUS$4.8	MUS$32.0	MUS$45.0	MUS$400.0
Life of equipment	6 years	7 years	25 years	25 years
Periodicity of decision	2 years	2 years	Once/LOM	Once/LOM
Lead time	1 year	1 year	2 years	3 years
Maximum purchase	100 units	15 units	1 unit	1 unit
Initial Capacity available E/N pits	100 / 40 units	14 / 6 units	-	-
Tonnage increment	2.9 Mt/unit	20.3 Mt/unit	5.0 Mt/unit	54.0 Mt/unit

The interaction between the different investment and operational alternatives within the mining complex's performance is shown in Figure 2. The left side of the figure presents a detail of plant LS's flow of material, where the blasting modes and the fleet investment alternatives affect both mines. These sources feed a possible new extra crusher, which in turn feeds the ball mill of LS, which can be aided by the addition of a secondary (2ry) crusher incorporated within the processing plant. This processing plant also has an operating mode that defines whether it will be operating at high throughput/ low recovery, or low throughput/ high recovery mode. The right side of the figure shows the global view of the adaptive mining complex, which consists on Figure 1, along with the set of alternatives mentioned (investment alternatives in red, mining modes in green, and processing modes in blue).

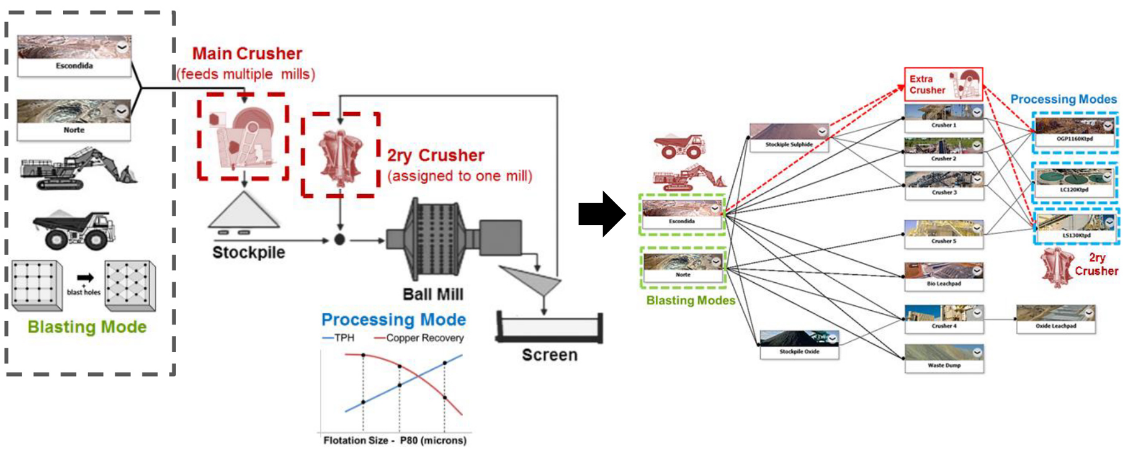

Figure 2: Detail of the interaction between investment and operating modes in the LS processing plant (left) and the global mining complex (right)

3.3 Results

Three cases are presented next, all of which consider geological uncertainty within the optimization in the form of equally probable simulations of the deposits. The first case corresponds to the traditional two-stage optimization where a fixed extraction and processing capacities are defined over the whole mining complex, as well as fixed operating modes. This is the traditional stochastic simultaneous optimization of a mining complex (Goodfellow and Dimitrakopoulos, 2016; Montiel and Dimitrakopoulos, 2018). Next, results obtained for the stochastic optimization considering alternatives is presented. In this case, the dynamic adaptive analysis is not performed, but the optimizer is allowed to define the optimal investment plan as in Goodfellow (2014), as well as the operational alternatives, similar to Montiel and Dimitrakopoulos (2015). Thereafter, the results are compared including the alternatives, both separately and simultaneously. Finally, the dynamic adaptive case is presented, which includes the possibility of branching over one-time investments, presenting a probabilistic solution of the life of asset design, which allows maintaining flexibilities available until more information is obtained to take the final design decisions.

3.3.1 Base case

The base case extraction plan and equipment purchase plan is presented in Figure 3, where there is a fixed equipment capacity for both Mine 1 (in black) and Mine 2 (in blue), even though the actual extraction (in dashed lines) is considerably less in various periods. In Figure 4, the risk profiles for the first Crusher, as well as for the three different plants are presented. As demonstrated, Crusher 1 (left of Figure 4) is working consistently at full capacity, however, there is still some capacity available at the processing plants. This analysis allows for identifying possible investments as interesting alternatives that could improve the performance of the mining complex, such as increasing the crushing capacity in order to increase the plants' feeds.

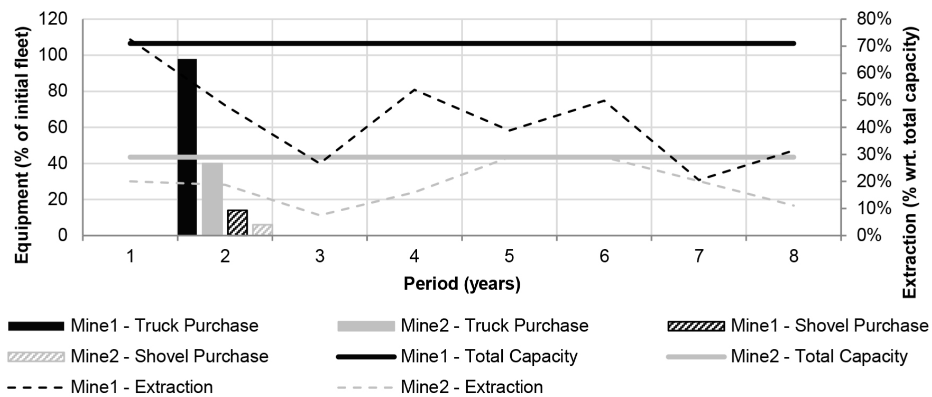

Figure 3: Base Case's mine extraction and fleet acquisition plan for Mine1: Escondida, and Mine2: Norte

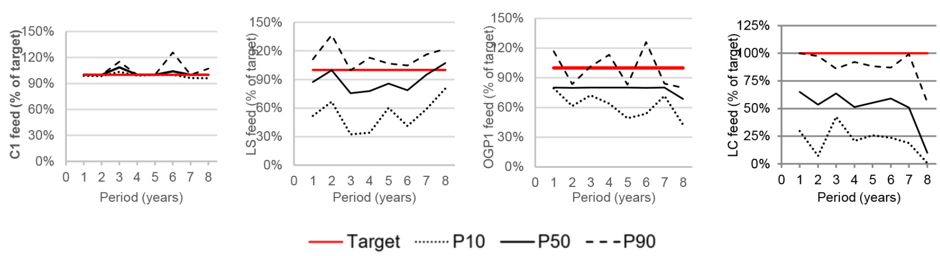

Figure 4: Base Case's risk profile of the annual material feed for i) Crusher 1, and plants ii) LS, iii) OGP1, and iv) LC, with respect to the target in red

3.3.2 Optimized alternatives

The second case presented includes the operating and investment alternatives mentioned in Figure 2, however, these alternatives are only considered as fixed strategic decisions in the two-stage optimization (as in Goodfellow (2014) investment acquisition plans and Montiel and Dimitrakopoulos (2015) operating mode alternatives). Figure 5 presents the annual mine extraction and equipment acquisition plan for both mines, showing the number of equipment units with respect to the initial quantity available in the left axis, and the percentage of extraction capacity (in full lines) and actually extracted (dashed lines) in the right axis, with respect to the total extraction capacity of the mining complex. In comparison to the base case fleet acquisition plan presented in Figure 3, the full alternative case reduces its initial fleet for the first four years, delaying the cost of investing in new trucks and shovels only for when this extra tonnage is required, instead of having idle equipment, as seen in the base case.

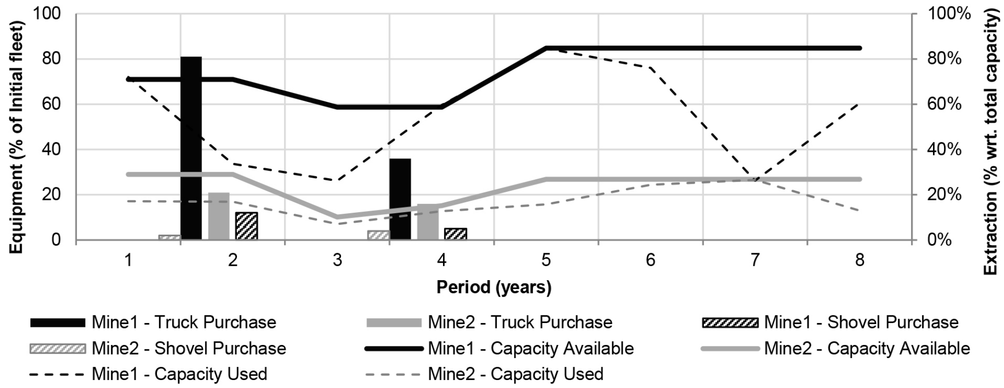

Figure 5: Full alternatives case's mine extraction and fleet acquisition plan for Mine1: Escondida, and Mine2: Norte pits

The risk analysis of the material fed into the main processing streams is presented in Figure 6. It can be seen in the leftmost graph that the capacity of Crusher 1 increases in year 5 due to an investment in an extra crusher in period 2, as this investment has a lead time of 3 years (Table 1). The targets of the three graphs on the right side of Figure 6 are modified by the acting operating modes, which adapt the plants' throughputs with an effect over their recovery (Table 2). Additionally, the capacity in the LS plant is increased in periods 6 through 8 because the plan chooses to invest in a 2ry crusher in period 4. Some deviations can be seen from the plant's maximum capacity, particularly in OGP1 and LC, however these are mostly during the final periods of the strategic plan.

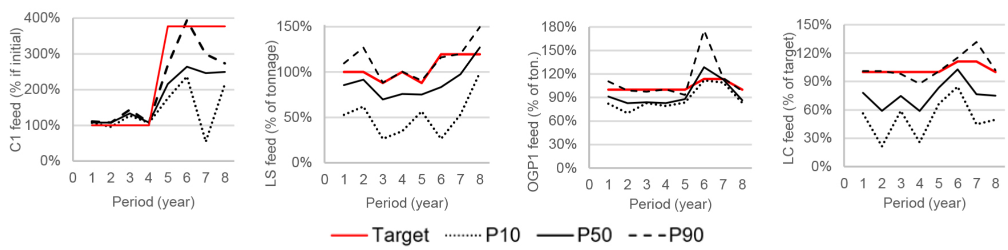

Figure 6: Risk profile of the Case with full alternatives for the annual material feed of i) Crusher 1, and plants ii) LS, iii) OGP1, and iv) LC, with respect to the target in red

Table 2: Operating mode's costs and effects information

Plant Operating Modes	Effect over Recovery	Effect over Throughput
LS Plant Mode	0.8%	-10%
LC Plant Mode	0.8%	-10%
OGP1 Plant Mode	0.9%	-12%
Mining Operating Modes	**Effect over Mining Cost**	Effect over Throughput
Escondida's Blast-hole Pattern	15%	7%
Norte's Blast-hole Pattern	10%	5%

3.3.3 Proposed adaptive dynamic case

The final solution tree representing the dynamic strategic plan with adaptive decisions is presented in Figure 7, where the first row illustrates the period of the plan, where $T = 8$, and the plan of period 1 corresponds to the same as the one presented in the previous section. The figure shows that, in this case, there is a 40% chance of branching over the investment of an extra crusher in period 2. Then, if the investment is done, there is a 30% chance of also investing in a 2ry crusher in period 3 and, if the extra crusher is not purchased in period 2, there is a 67% chance of investing in it later in period 4. If this is done, there is also a 50% chance of also investing in a 2ry crusher in that same period. With these probabilities, it is possible to calculate the probability of each branch, showing that there is only a 20% chance of not investing in any CAPEX alternative (last branch), but there is an 80% chance of investing in the extra crusher between periods 2 and 4, showing that it might be interesting to advance this investment, compared to the fixed case with alternatives presented in the previous section.

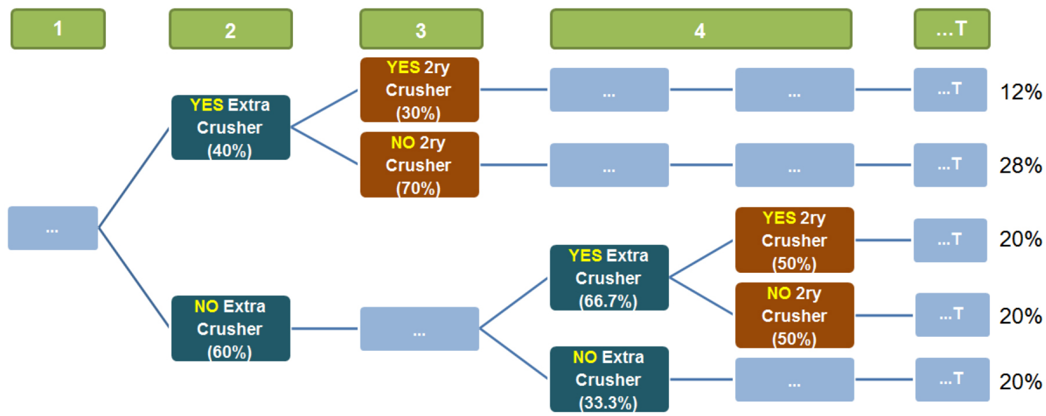

Figure 7: Investment solution tree representation of the strategic adaptive plan

Each branch of this solution tree includes a full production schedule, with its corresponding operating modes and minor equipment purchases. For example, the extraction and equipment purchase program for the second branch, which has the highest probability of occurring (28%), is presented in Figure 8, where it shows that, in this case, the overall fleet size is reduced compared to the original base case. However, in this case, more trucks are purchased in period 2 (40 instead of 45 as in the previous case for Escondida and 21 instead of 14 for Norte) and there is also an increase in extraction capacity towards the last years, investing in trucks and shovels during years 4, 5, and 6 in both mines. The material feed for the extra crusher purchased in period 2, as well as for the three different plants is presented in Figure 9. The figure also shows the effect of the different operating modes over the processing capacities (red line), compared to the initial targets (dotted-line), where it can be seen that the risk profiles are able to closely follow the capacities adapted by the operating modes, with some slight deviations.

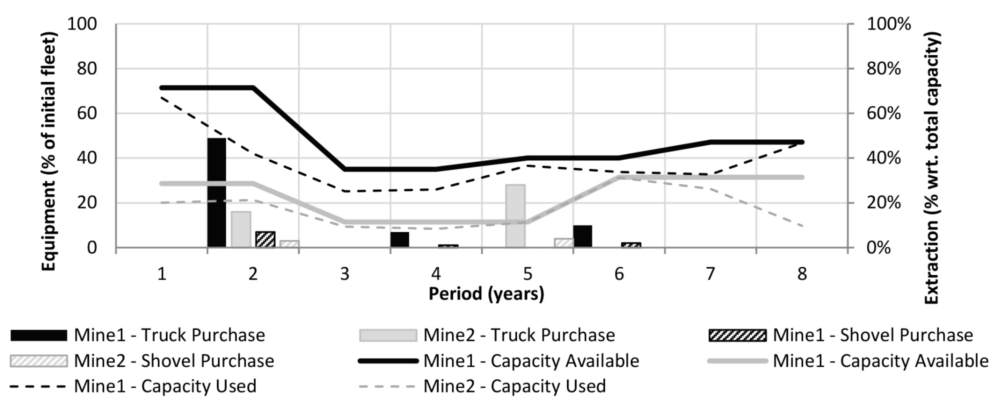

Figure 8: Mining extraction and fleet acquisition plan for both mines, under the second branch of the dynamic adaptive model

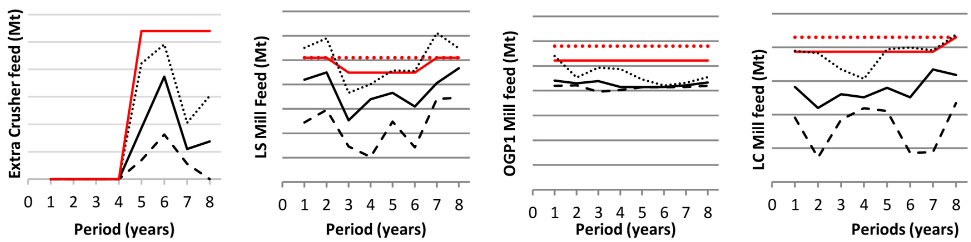

Figure 9: Adaptive plan's risk profile for branch 2 of the annual material feed for i) Crusher 1, and plants ii) LS, iii) OGP1, and iv) LC, with respect to the operating mode target (continuous red), and to the original target without operating modes (dotted red)

3.4 Discussion

The NPV distribution for each of the three cases presented is shown in Figure 10, where the horizontal axis presents the scaled NPV with respect to the original base case, and the vertical axis presents the probability of obtaining at most that NPV. The values have been scaled to the 50^{th} percentile of the initial base case (i.e., the P50 value) for confidentiality reasons, showing that if investment and operational alternatives are included ("2-STAGE WITH ALT." curve in dashed grey), the NPV of the mining complex can increase between 8 and 12% compared to the original base case ("2-STAGE WITHOUT ALT." curve in light grey). This difference is due to three main reasons: first, the expansion opportunities obtained by allowing the processing streams to expand their crushing capacities; second, because of the flexibilities that the operating modes provide to the configuration of the different component, being able to have a better control over how the material is being processed according to its characteristics; finally, because a lot of this value is obtained by optimizing the timing of purchase of the different equipment, it delays investments that are not immediately required. Figure 10 also shows the NPV distribution for the dynamic adaptive case proposed. Results show that the dynamic analysis provides a more general look at the mining complex's future performance, maximizing the value of possible opportunities, while hedging from risk.

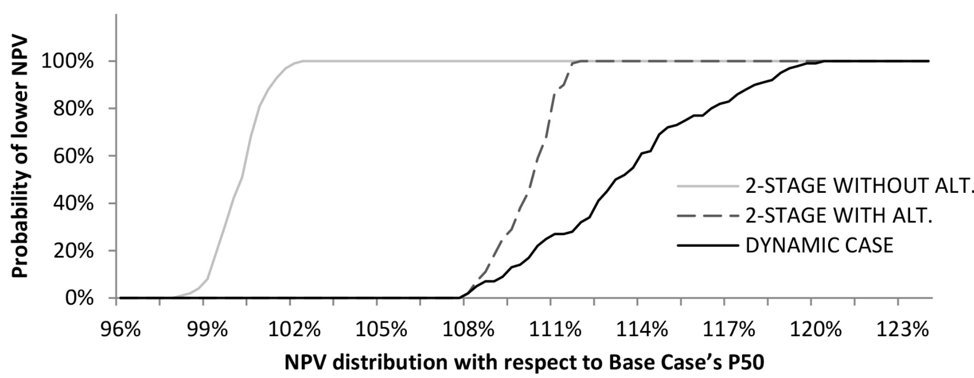

Figure 10: Net present value cumulative probability distribution for the three cases presented in Section 3.3

The strong differences in NPV are directly related to the significant differences in terms of the physical extraction schedules obtained from each optimization, meaning that these different optimization models produce mine plans that choose to extract different areas and amounts of material in different periods. This can be seen in Figure 11, which presents a comparison between the schedule of Mine 1 for the traditional two-stage optimization with alternatives (Section 3.3.2), and the proposed dynamic analysis (Section 3.3.3). It can be seen that the first two periods are equal between all schedules, and the third one is common between the base case and branches 3,4, and 5, following the branching schedule presented in Figure 7.

It is interesting to notice how the schedule corresponding to branch 5 is clearly smaller than the one obtained by the two-stage method, showing the extent of the effect that these investments have over the optimal schedule of the deposit.

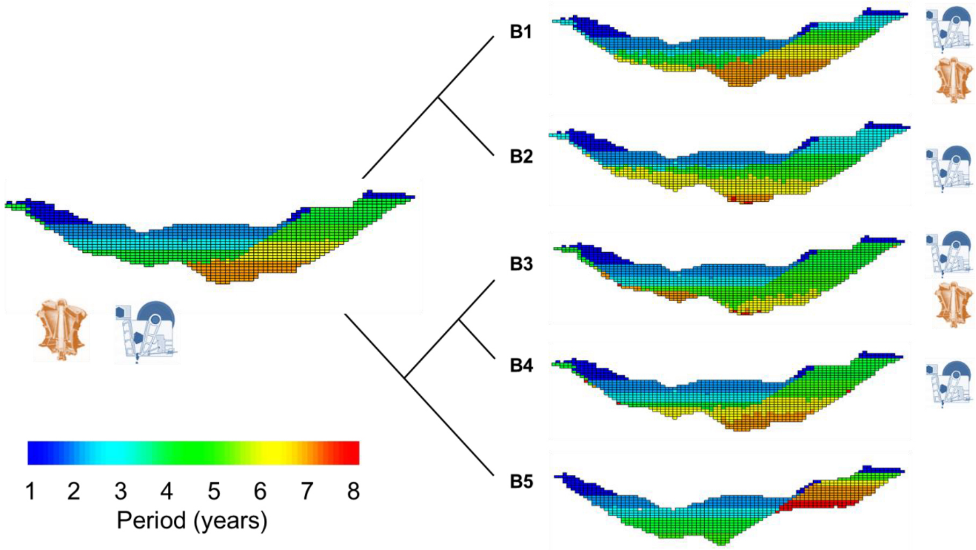

Figure 11: Comparison between two-stage fixed schedule (left) and adaptive schedules per branch (right), showing the corresponding investments in each case

4 Conclusions and future work

In conclusion, the current application of the dynamic optimization over the Escondida mining complex has shown that the proposed method is able to capitalize on the full extent of the information provided by the set of geological simulations used to represent the deposit's uncertainty, as these provide an understanding of the deposit's areas where the variability and/or the lack of data may cause the strategic plan to change. The method proposed in this paper is able to produce feasible strategic plans that are operational in the short-run, and can take advantage of possible future opportunities, allowing the mining complex to be prepared for the possible effects of uncertainty over the strategic plan, being able to react in a timely manner.

The method proposed is based on the two-stage stochastic optimization of mining complexes and uses multistage optimization techniques to represent the strategic plan as a scenario tree with representative branching. This means that transition probabilities between one stage and the next are filtered to ensure that they are representative and are iteratively solved as two-stage optimizations over the corresponding scenarios, avoiding overfitting problems and providing a probabilistic approach to the possible developments of the mineral value chain.

The algorithm was able to solve the Escondida mining complex comprised of almost 200,000 blocks contained in two mines, with six different processing streams and stockpiles. Results showed that the proposed analysis can increase project value by between 8% and 20% compared with the traditional fixed two-stage plan. This value is mostly created due to the optimized investment timings, the possibility of expansions, the better configuration of processing streams obtained through the different operating modes, and the ability to branch the strategic plan, allowing the optimizer to consider and develop alternatives that might be profitable in the future.

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